

From Market Opening to Innovation: Effective Contestability in Digital Markets

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Abstract

When does opening a bottleneck in digital markets induce genuine contestability, and what would the consequential outcomes be? To answer these questions, I develop a two-period stochastic general equilibrium model in which a dominant platform and a potential rival interact strategically. Nesting this game within the general equilibrium connects market-level competition to aggregate production and welfare.

Effective market access determines how much potential demand a rival can obtain, and the resulting commercial scale determines whether entry and innovation are privately viable. Fixed entry costs create a commercially viable threshold while scale-dependent innovation returns generate a second, higher threshold for rival innovation. These thresholds separate the market into three regimes: no rival entry, rival entry without innovation, and rival entry with innovation.

Applying the model to EU iOS mobile app distribution under the Digital Market Act shows that post-DMA observed effective market access κ_1^{obs} crosses the commercial-entry threshold κ^E and supports rival entry. However, full effective market access still leaves the rival trapped in an "entry without innovation" regime with a marginal benefit-to-cost ratio of 0.949. Diagnostics show that closing 22.62% of rival inherited quality and productivity gaps lowers rival innovation threshold to the observed access level. Greater access reduces platform profits but stimulates platform innovation. Aggregate consumption-equivalent welfare increases (0.222%–0.470%) via downstream linkages. These robust findings demonstrate that rival entry precedes innovation, and platform innovation and aggregate welfare gains remain positive, illustrating that formal compliance, commercial viability, and dynamic contestability are distinct sequential tests of regulatory success.

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1 Introduction

Opening a digital market is not the same as introducing an effective competitor to the market. A legal rule or regulation may grant a rival the formal right to reach users, but factors, like interface design, interoperability, and switching costs, often restrict users' actual adoption. Even when market access is possible, the potential rival might still not enter if there is insufficient market demand to offset fixed entry costs. Moreover, an active rival might lack the commercial scale required to support continued innovation. The central question of this paper is therefore not whether a digital market bottleneck can be opened, but whether that opening can be converted into a commercially viable entry and sustained dynamic competition.

The same features that make them valuable—network effects, economies of scale and scope, switching costs, accumulated data, and control over distribution channels—can also make an established platform difficult to challenge (Rochet and Tirole, 2003; Armstrong, 2006; Biglaiser and Crémer, 2020). A small rival may face not only a current route-to-market problem, but also competitive asymmetries accumulated while that route was restricted. Because digital services can also serve as gateways and productive inputs for activity elsewhere in the economy, competition at a digital bottleneck can affect more than the price and market share of the digital service itself.

The European Union's Digital Markets Act (DMA) is a leading example of the shift toward ex-ante digital market regulation (European Parliament and Council of the European Union, 2022). The DMA pursues contestability and fairness through a wide set of obligations. One important channel is the opening of gatekeeper-controlled routes that allow rivals to reach users. But a formal entitlement to use a route to market does not by itself determine whether users can discover, adopt, and continue using a rival's service/product, or whether the resulting commercial base can support rival innovation. The same economic issue arises when competition authorities use interoperability, non-discrimination, steering, or access remedies in other settings.

Existing research has laid an important foundation for solving this puzzle. The platform literature investigates how network effects shape entry and pricing (Rochet and Tirole, 2003; Armstrong, 2006). Research on digital competition policy examines why conventional enforcement may operate slowly in markets prone to tipping and analyzes the design and implementation of ex-ante obligations (Crémer et al., 2019; Cabral et al., 2021; Bostoen, 2023; Cennamo et al., 2023). The competition and innovation literature shows that stronger competition can raise or lower innovation incentives depending on market structure, innovation rents, and the relative positions of incumbents and entrants (Aghion et al., 2005; Acemoglu and Cao, 2015; Akcigit and Kerr, 2018). Some other work studies regulation as a determinant of innovation. Labor regulation can reduce firms' propensity to innovate and weaken their innovation response to demand shocks, while policy-induced price incentives can redirect technical change across technologies (Aghion et al., 2023, 2016). Waldfogel (2024) and Fang and Kim (2026) also show that regulatory scrutiny and access obligations can alter gatekeeper conduct and downstream competition.

What remains less explained is when a change in market access can induce a sustainable competitive process. Legal compliance or changed platform conduct does not reveal whether a rival can acquire enough users and revenue to enter, scale, and innovate. On the contrary, a small rival share does not by itself reveal whether growth is still constrained by current market access frictions or by an inherited competitive position. The DMA impact assessment identified weak contestability, unfair practices, and reduced innovation as central problems (European

Commission, 2020), but it did not provide a structural mapping from market access to rival demand, entry, innovation and welfare.

This paper develops that mapping. The model shows when opening a digital market bottleneck and improving market access can translate into a rival’s commercially viable entry, and when the resulting commercial scale can make rival innovation privately profitable. Greater effective market access expands the demand that a potential rival can convert into a market foothold. This commercial base determines whether entry covers its fixed entry costs and whether successful innovation generates a sufficiently large private return. The paper does not assume that access automatically produces entry nor that entry automatically produces innovation. It derives the conditions under which each transition occurs.

The model developed in this paper tries to distinguish between two objects that are often combined in the discussion of digital market contestability—*effective market access* and *inherited competitive state*. The former refers to the rival’s ability to reach users and convert potential demand into realized demand. The latter is the quality, productivity, and other competitive conditions with which the rival enters the market. It does not assume that inherited state developed independently of earlier restrictions on market access. Instead, these inherited conditions may summarize accumulated disadvantages in scale, networks, data, and reputation due to the cumulative effects of the preceding market access restrictions. This separation allows the model to distinguish the removal of a route-to-market disadvantage from the slower process through which firms build capabilities and commercial positions.

The interaction between the inherited state and effective market access is embedded in a two-period stochastic model with a dominant platform and a potential rival. The inherited market is platform-only. After observing period-1 effective market access, the rival decides whether to enter. The platform and an active rival then choose investments in incremental and disruptive innovation. Innovation success changes both consumer-facing quality and production productivity. This game in the digital sector is nested inside a general equilibrium in which digital products are consumed by households and used as intermediate inputs in non-digital production. Therefore, rival entry and innovation affect the digital price index, labor demand, wages, household income, non-digital production, and welfare. These aggregate outcomes, in turn, affect the profitability of rival entry and innovation. The framework combines strategic market structure and innovation with an economy-wide consistency condition (Atkeson and Burstein, 2008; Neary, 2016; Pellegrino, 2025; Baqaee and Farhi, 2019).

This framework delivers five main implications.

First, a rival’s commercialization is jointly determined by effective market access and inherited competitive position. Greater effective market access directly scales a rival’s market share, allowing an active rival to convert more potential demand into realized sales, although the expansion diminishes. However, the level of commercialization achieved at any given market access depends on the inherited state the rival enters with. Full market access removes the modeled market access disadvantage, but it does not eliminate inherited differences in product attractiveness or imply equal market shares.

Second, better commercialization opportunities do not automatically lead to rival entry. Fixed entry costs create a commercial entry threshold (κ^E). Below this threshold, improved market access may increase the demand the rival could capture if it entered, but the commercialization value is not enough to cover the fixed entry cost. The rivals’ realized market share remains zero since entry is not economically viable. At the threshold, commercialization value equals fixed entry costs, and entry becomes privately attractive. Beyond the threshold, further improvements in market access could continue expanding the active rival’s commercial scale.

Third, commercially viable entry does not necessarily produce dynamic competition. Rival innovation becomes privately profitable only when the commercial base is large enough for the expected gain from successful innovation to cover its marginal cost. This generates a second threshold, κ^I , above the commercial-entry threshold. These two thresholds separate the market into three different regimes: no rival entry (Regime I); rival entry without innovation (Regime II); and rival entry with innovation (Regime III). A market can thus support an active rival without yet supporting its innovation.

Both thresholds depend on the conditions inherited at the time of market regulation. Rival with stronger inherited quality, productivity, or innovation capacity can enhance its commercialization value and lower the effective market access required for rival entry and innovation. Therefore, effective market access and inherited competition state are complementary in the transition from market opening to dynamic competition.

Fourth, the effect of greater effective market access on platform innovation is theoretically ambiguous. Improved market access shifts digital expenditure toward the rival and reduces the platform's market share and profit. This business-stealing effect might weaken the platform's incentive to innovate. Meanwhile, lower quality-adjusted digital prices can expand total digital expenditure and therefore change the profit gain associated with a successful platform innovation. So platform profit and its R&D need not move in the same direction.

Fifth, improved market access has direct price and production benefits, but the complete aggregate welfare effect cannot be signed analytically. Conditional on rival activity and fixed price-quality state, greater access lowers the quality-based digital price. At the same time, the effect of a lower digital price index has also been propagated by the channel of digital products as intermediate inputs in the non-digital production, raising labor productivity and equilibrium wage. However, aggregate welfare also depends on changes in labor reallocation, innovation expenditures, firm profits, and foreign ownership. These forces need not all work in the same direction. The sign and the magnitude of the complete welfare effect are results of quantitative general equilibrium rather than unconditional theoretical predictions.

For quantitative analysis, the framework is applied to iOS mobile app distribution in the European Union. This setting clearly provides an identified bottleneck in that specific digital market. The observed post-DMA effective market access is $\kappa_1^{\text{obs}} = 0.47$.¹

The benchmark calibration yields a commercial entry threshold of $\kappa^E = 0.2342$. The observed market lies in Regime II, where rival entry is commercially viable, but rival innovation is not. Holding the inherited competitive state fixed, full market access raises the rival's market share to 4.1% but still does not enable rival innovation. Therefore, removing the remaining market access gap alone does not produce dynamic competition.

Two complementary exercises show how the inherited rival position and rival innovation capacity change this conclusion. Along a path that narrows the inherited gaps between the rival and the platform in quality and productivity, if we can close 22.62% of each gap, it can bring the innovation threshold down to the observed market access κ_1^{obs} , even though the rival's market share is only 4.29%. In another recalibrated path with stronger rival innovation capacity, the innovation threshold falls to $\kappa^I = 0.4284$, shifting the observed market to Regime III. These results show that dynamic competition can be introduced and operate well before equal market shares are realized. They also demonstrate that markets with similar observed entry and market shares may have very different innovation prospects.

The paper makes three contributions. First, it contributes to a theory of the transition from

¹A companion measurement framework disciplines the model's effective market access (Cheng et al., 2026).

market opening to dynamic competition. Existing platform and digital regulation research explains how market bottlenecks developed and how access obligations may alter players' conduct. This paper delivers two distinct thresholds to explain why legal opening, entry, market share, and innovation can provide different assessments of the same reform.

Moreover, this paper contributes methodologically by integrating endogenous entry and strategic innovation into a general equilibrium model. The inner firm game determines market structure and R&D choices. The outer equilibrium makes demand, wages, income, production, and resource use consistent with those decisions. This structure permits the same model to study commercial viability, rival and platform innovation incentives, downstream production effects, and welfare.

Last but not least, this paper provides a quantitative framework for separating effective market access from the competition state inherited at the time of regulation. The empirical application uses measured market access conditions and observed market outcomes to locate the actual market relative to the theoretical entry and innovation thresholds. The counterfactuals show how stronger inherited conditions or innovation capacity could change those thresholds. This offers a way to empirically evaluate whether a bottleneck-opening intervention has produced only formal/legal access, a commercially viable rival, or a dynamically innovative competitor.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 derives the economic mechanisms of the model. Section 4 and Section 5 apply the model to mobile app-distribution application market, describing the calibration and counterfactual design. Section 6 presents the quantitative and sensitivity results. Section 7 discusses policy implications, and then Section 8 concludes.

2 A Model of Effective Market Access, Entry and Innovation

Digital-market regulation can affect contestability through several channels. This paper isolates one foundational mechanism: whether a potential challenger can reach users through a gatekeeper-controlled bottleneck and convert potential demand into realized sales. Usable market access is necessary for this commercialization opportunity, but positive demand is not sufficient to make entry privately viable. Entry is commercially viable only when the discounted expected value of operating in the market, net of the challenger's optimally chosen innovation expenditure, is sufficient to cover the fixed cost of entry. Even after entry, the resulting commercial base may remain too small to make innovation privately profitable. The model therefore distinguishes three outcomes that are often conflated: effective market access, commercially viable entry, and innovation supported by commercial scale.

The model formalizes this sequence in a two-period stochastic general equilibrium economy. An improvement in effective access increases the demand that the challenger could serve if it entered, thereby expanding the commercial opportunity associated with entry. The challenger enters only when the value of that opportunity crosses a commercial-viability threshold. Conditional on entry, further improvements in access expand the challenger's realized sales base. Innovation becomes privately profitable only if that commercial base is sufficiently large for the expected gain from successful innovation to cover its cost. The model can generate two distinct thresholds: one for commercially viable entry and a second for challenger innovation. Whether either threshold is crossed depends jointly on the inherited competitive conditions, the fixed cost of entry, and firms' innovation technologies. Entry and innovation then affect digital prices, labor demand, firm profits, household income, non-digital production, and welfare.

The model is not intended as a general model of the DMA or of every channel through which digital-market regulation may affect fairness and contestability. It isolates the market-access mechanism through which some digital-market interventions operate and studies its interaction with inherited competitive conditions and endogenous innovation.

2.1 Economic Environment And Timing

Consider a two-period general-equilibrium economy, indexed by $t \in \{0, 1\}$. Period 0 describes the inherited market environment before the introduction of digital market regulation and the transition during which entry and innovation expenditures are undertaken. Production in this period uses inherited technologies, and period-0 resources finance household consumption as well as any entry and innovation expenditures. Period 1 describes the economy after the regulations have been enforced and innovation outcomes have occurred. The two-period structure allows firms to incur entry and innovation costs during the transition and receive the resulting commercial and technological returns in the subsequent period.

The economy consists of a representative household H , a dominant digital platform P , a potential challenger/challenger R , and a competitive non-digital sector N . A policy, implementation, compliance, and adoption process produces the period-1 effective-access state κ_1 . The model does not represent this process as a strategic actor or assume that a regulator directly chooses κ_1 .²

The representative household supplies $\bar{L}$ units of labor inelastically consumes a non-digital numeraire good C_t and digital products or services D_t in each period. Digital services supplied by the dominant platform and challenger are imperfect substitutes. The household owns the firms while a fraction $\phi \in [0, 1]$ of the dominant platform is foreign-owned. It receives labor income, challenger profit, and the share $1 - \phi$ of the platform's net distributions; the remaining share is remitted abroad.³

The inherited digital market is served only by the dominant platform. A potential challenger may enter after observing the period-1 effective access state. I therefore maintain throughout that $e_{R0} = 0$. This assumption isolates the response at the center of the analysis: whether an improvement in effective market access makes challenger entry commercially viable. The model abstracts from the continuation and exit decisions of challengers that are already active in the inherited market. However, conditional on entry, the challenger's commercial scale and innovation choices respond endogenously to period-1 market conditions.

The dominant platform and active rival can invest in two directions of innovation. Incremental innovation improves the existing digital service, whereas disruptive innovation can generate a larger technological or product-quality improvement. Both firms can undertake either type of innovation, although they may possess different comparative advantages and innovation efficiencies across the two directions.

The economy begins with the inherited state

$$\mathcal{S}_0 = \{\kappa_0, \mathcal{H}_0\}, \quad \mathcal{H}_0 \equiv \{q_{P0}, q_{R0}, A_{P0}, A_{R0}, B_0\}, .$$

Here κ_0 is inherited effective market access facing the potential rival. Because the challenger is not active in period 0, κ_0 does not directly enter period-0 household or intermediate demand.

²The regulator sets regulatory obligations. These obligations do not directly determine market shares or firm entry. Instead, they may change effective access from κ_0 to κ_1 .

³Thus, $\phi = 0$ corresponds to complete domestic ownership of the platform and $\phi = 1$ to complete foreign ownership.

Instead, it describes the inherited access environment facing a potential entrant and provides the period-1 access state under the no-improvement counterfactual.

The variables q_{P0} and A_{P0} denote the platform's inherited service quality and production productivity. The variables q_{R0} and A_{R0} denote the pre-innovation quality and productivity with which the potential challenger would enter the period-1 market. Because the challenger is inactive in period 0, these are latent pre-entry characteristics rather than realized period-0 market outcomes. Finally, B_0 denotes inherited non-digital productivity.

The vector $\mathcal{H}_0$ collects the inherited economic and competitive conditions other than the inherited effective-access state κ_0 . Entry costs and the parameters governing firms' innovation technologies are structural primitives rather than elements of $\mathcal{H}_0$. Once κ_1 is realized, entry and innovation decisions depend on κ_1 , $\mathcal{H}_0$, and these structural primitives. The inherited access state κ_0 characterizes the initial access environment and provides the no-improvement benchmark.

This analytical separation does not imply that $\mathcal{H}_0$ developed independently of earlier access conditions. Historically restricted access may have affected the challenger's opportunity to acquire users, develop networks, collect data, improve its product, build a reputation, or accumulate other competitive capabilities. The model conditions on $\mathcal{H}_0$ at the transition date without identifying the historical process that produced it.

The timing is as follows:

1. **Inherited market environment.** The economy begins from the pre-regulated state $\mathcal{S}_0$. The period-0 digital market is served only by the dominant platform, and production takes place using the inherited platform and non-digital technologies. The potential challenger has the latent pre-entry characteristics (q_{R0}, A_{R0}) but is not part of the period-0 household or producer choice sets.
2. **Realization of period-1 effective access.** The policy and implementation process produce the period-1 access state κ_1 . Under the no-improvement counterfactual, $\kappa_1 = \kappa_0$. Under an access-improving intervention, $\kappa_1 > \kappa_0$. Firms observe κ_1 before making entry and innovation decisions.
3. **Entry.** After observing κ_1 , the potential challenger decides whether to enter the period-1 digital market, given the inherited economic and competitive conditions $\mathcal{H}_0$. Let $e_R \in \{0, 1\}$ denote this decision. Entry requires a fixed resource cost $F_R > 0$.
4. **Innovation investment.** Conditional on the entry decision, the platform and the active rival simultaneously choose investments in incremental and disruptive innovation. Denote these choices by

$$x_{jk}(\kappa_1), \quad j \in \{P, R\}, \quad k \in \{I, D\}.$$

The entry cost and innovation expenditures are paid in units of the period-0 non-digital numeraire. They therefore reduce the period-0 resources available for household consumption.

5. **Innovation realization.** Stochastic innovation outcomes are realized between periods. These outcomes determine firms' period-1 qualities and productivity.
6. **Period-1 equilibrium.** In period 1, firms produce with the resulting technologies. Current effective access affects rival's ability to commercialize its services, while inherited

conditions and realized innovation affect firms' relative price-quality positions and production costs. Households make their consumption choices, firms employ labor and earn profits, and all markets clear.

2.2 Effective Market Access and the Inherited Competitive State

Let $\kappa_t \in [0, 1]$ denote the rival's current *effective market access*: its usable ability to reach users and translate potential demand into realized commercial demand. The dominant platform's market access weight is normalized to one. Conditional on rival activity, the rival's corresponding market access weight is κ_t .

Thus, $\kappa_t = 0$ means that the rival is commercially inaccessible in the modeled demand system, whereas $\kappa_t = 1$ means that the platform and rival receive equal access weights, conditional on their prices and qualities. I refer to $\kappa_t = 1$ as the *full access benchmark*. Full market access removes the contemporaneous access disadvantage represented by κ_t ; it does not imply equal products, equal qualities, equal productivities, equal market shares, homogeneous-good Bertrand competition, or the disappearance of networks, data, reputation, and other competitive asymmetries inherited at the transition date.

The distinction between κ_t and $\mathcal{H}_0$ is analytical rather than causal. The model conditions on the inherited state when evaluating a change in current market access, but it does not assume that the inherited state is independent of past contestability. Historically restricted market access may have limited a challenger's ability to acquire users, benefit from network effects, collect data, attract complementary suppliers, learn from market participation, finance product development, or establish a reputation. Those cumulative effects may be reflected in inherited quality, productivity, market presence, or innovation capacity.

The baseline model does not specify the longer-run law of motion through which past market access, commercial scale, network formation, data accumulation, and repeated innovation jointly produce the inherited state. It therefore does not identify whether an observed competitive asymmetry originates in intrinsic product differences, earlier exclusion, cumulative scale advantages, consumer preferences, superior investment, or some combination of these forces. The counterfactual object is consequently the period-1 equilibrium generated by alternative current market access states conditional on inherited conditions,

$$\mathcal{Y}_1(\kappa_1 \mid \mathcal{H}_0),$$

where $\mathcal{Y}_1$ collects equilibrium entry, innovation, allocations, profits, and welfare. The full-market-access benchmark is therefore a conditional boundary exercise, not a prediction of the long-run market structure that would emerge after many periods of capability accumulation under permanently open market access.

Effective market access does not directly determine entry, market share, quality, productivity, or innovation. It changes the commercial opportunity available to an active challenger. Whether that opportunity supports entry or innovation depends jointly on κ_1 , the inherited state, relative prices and qualities, entry and innovation costs, firms' innovation efficiencies, and general-equilibrium conditions.

The model does not assume that a regulator directly chooses κ_1 . Particular rules, enforcement decisions, implementation practices, platform responses, technical conditions, and user or business adoption may jointly produce the realized market access state. The threshold results derived in Section 3 therefore identify the effective market access conditions required for entry or innovation, not the legal strictness or enforcement expenditure required to produce those

conditions. The empirical construction used to discipline κ_t is introduced only after the model's theoretical implications have been established.

2.3 Household Preferences

A representative household lives for two periods and supplies $\bar{L}$ units of labor inelastically in each period. It consumes a non-digital good C_t and digital services supplied by the platform and rival, denoted by d_{Pt} and d_{Rt} , respectively. Rival consumption is the consumption of services supplied by the rival, conditional on its activeness in the market.⁴

Lifetime expected utility is

$$\mathcal{U} = \log X_0 + \beta \mathbb{E}_0 [\log X_1], \quad \beta \in (0, 1), \quad (1)$$

where X_t is a consumption composite, the expectation is taken over period-1 innovation outcomes, and β is the discount factor.

The consumption composite combines non-digital consumption and digital services:

$$X_t = \left[(1 - \mu) C_t^{\frac{\sigma-1}{\sigma}} + \mu D_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \mu \in (0, 1), \quad (2)$$

where μ governs the weight on digital services, and $\sigma > 0$ is the elasticity of substitution between digital and non-digital consumption.

When the rival is active, the digital-services composite D_t combines quality-adjusted services from the dominant platform and the rival:

$$D_t = \left[(q_{Pt} d_{Pt})^{\frac{\sigma_D-1}{\sigma_D}} + \kappa_t^{\frac{1}{\sigma_D}} (q_{Rt} d_{Rt})^{\frac{\sigma_D-1}{\sigma_D}} \right]^{\frac{\sigma_D}{\sigma_D-1}}, \quad \sigma_D > 1, \quad (3)$$

where $q_{jt} > 0$ denotes the quality of the service supplied by the platform or the rival $j \in \{P, R\}$ and σ_D is the elasticity of substitution between the two digital services. Effective market access κ_t enters directly as the rival's relative commercial-access weight. The dominant platform's corresponding weight is normalized to one.

If the challenger is inactive, it is excluded from the household's choice set:

$$d_{Rt} = 0, \quad D_t = q_{Pt} d_{Pt}. \quad (4)$$

The non-digital good is the numeraire. Digital services are sold at predetermined prices $P_P > 0$ and $P_R > 0$, which are held fixed across periods and counterfactuals. This assumption allows the analysis to focus on access, commercialization, and innovation rather than endogenous pricing.

The household's period- t budget constraint is

$$C_t + P_P d_{Pt} + P_R d_{Rt} = I_t. \quad (5)$$

Let $\mathbf{x}_j \equiv (x_{jI}, x_{jD})$. The household owns the firms, and the competitive non-digital sector earns zero profit, so the household's disposable income is

$$I_t = w_t \bar{L} + (1 - \phi) \pi_{Pt} + \pi_{Rt} - \mathbf{1}\{t = 0\} [(1 - \phi) \mathcal{C}_P(\mathbf{x}_P) + e_R (F_R + \mathcal{C}_R(\mathbf{x}_R))]. \quad (6)$$

⁴So rival consumption is zero when the rival is inactive.

Here, w_t is the wage, π_{jt} denotes the operating profit of digital supplier j and $\pi_{Rt} = 0$ when the challenger is inactive. $C_j(\mathbf{x}_j)$ denotes the digital supplier j 's investment in incremental innovation I and disruptive innovation D . The final term accounts for the domestically financed period-0 resources used for entry and innovation. Foreign owners finance the share ϕ of platform innovation expenditure, consistent with the foreign-remittance accounting below.

Define the quality-adjusted prices of the two digital services as

$$p_{Pt} \equiv \frac{P_P}{q_{Pt}}, \quad p_{Rt} \equiv \frac{P_R}{q_{Rt}}. \quad (7)$$

Conditional on challenger activity, the unit-expenditure index for the digital composite is

$$P_{Dt} = \left[p_{Pt}^{1-\sigma_D} + \kappa_t p_{Rt}^{1-\sigma_D} \right]^{\frac{1}{1-\sigma_D}}. \quad (8)$$

The shares of digital expenditure allocated to the challenger and platform are

$$s_{Rt} = \frac{\kappa_t p_{Rt}^{1-\sigma_D}}{p_{Pt}^{1-\sigma_D} + \kappa_t p_{Rt}^{1-\sigma_D}}, \quad s_{Pt} = 1 - s_{Rt}. \quad (9)$$

When the challenger is inactive, $s_{Rt} = 0$ and $s_{Pt} = 1$.

Equivalently, relative demand satisfies

$$\frac{d_{Rt}}{d_{Pt}} = \kappa_t \left(\frac{q_{Rt}}{q_{Pt}} \right)^{\sigma_D-1} \left(\frac{P_P}{P_R} \right)^{\sigma_D}. \quad (10)$$

Equation (10) displays the two separate determinants of rival demand. Effective market access determines whether and to what extent the rival can reach users, while relative quality determines how attractive its service is once access is available. However, we should note that this is an accounting decomposition inside the model, not a causal decomposition of the rival's observed market position. In particular, the inherited quality term may partly embody scale, network, data, reputation, or product-development differences accumulated under the preceding market regime.

Under the raw CES weights in the equation (2), the price index for total consumption is

$$P_{Xt} = \left[(1 - \mu)^\sigma + \mu^\sigma P_{Dt}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (11)$$

The household's digital expenditure share is

$$S_t^H = \frac{\mu^\sigma P_{Dt}^{1-\sigma}}{(1 - \mu)^\sigma + \mu^\sigma P_{Dt}^{1-\sigma}}. \quad (12)$$

Optimal expenditure allocations satisfy

$$C_t = (1 - S_t^H) I_t, \quad (13)$$

$$d_{Pt} = \frac{S_t^H (1 - s_{Rt}) I_t}{P_P}, \quad (14)$$

$$d_{Rt} = \frac{S_t^H s_{Rt} I_t}{P_R}. \quad (15)$$

Optimal composite consumption in period t is

$$X_t^* = \frac{I_t}{P_{Xt}}. \quad (16)$$

Household welfare therefore changes through both real income and the quality-adjusted cost of consumption.

2.4 Non-Digital Market

The non-digital good sector produces competitively using labor and digital services as intermediate inputs:

$$Y_{Nt} = B_t L_{Nt}^{1-\iota} \tilde{D}_t^\iota, \quad \iota \in (0, 1), \quad (17)$$

where $B_t > 0$ denotes non-digital productivity, L_{Nt} is labor, and $\tilde{D}_t$ is a composite of digital intermediate inputs. The parameter ι is the digital-input expenditure share.

Let m_{Pt} and m_{Rt} denote the quantities of dominant-platform and rival-platform services used as intermediate inputs. Conditional on rival activity, the digital-input composite is:

$$\tilde{D}_t = \left[(q_{Pt} m_{Pt})^{\frac{\sigma_D-1}{\sigma_D}} + \kappa_t^{\frac{1}{\sigma_D}} (q_{Rt} m_{Rt})^{\frac{\sigma_D-1}{\sigma_D}} \right]^{\frac{\sigma_D}{\sigma_D-1}}. \quad (18)$$

The same effective market access weight κ_t is used for household and intermediate demand. This means that the market conditions governing usable access affect household and business adoption symmetrically.⁵

If the rival is inactive,

$$m_{Rt} = 0, \quad \tilde{D}_t = q_{Pt} m_{Pt}. \quad (19)$$

Taking wages, digital-service prices, qualities, and effective access as given, the non-digital producer solves

$$\max_{L_{Nt}, m_{Pt}, m_{Rt} \geq 0} \left\{ B_t L_{Nt}^{1-\iota} \tilde{D}_t^\iota - w_t L_{Nt} - P_P m_{Pt} - P_R m_{Rt} \right\}. \quad (20)$$

Cost minimization within the digital-input composite produces the same within-digital expenditure shares as household demand. Conditional on the rival's activity,

$$\frac{m_{Rt}}{m_{Pt}} = \kappa_t \left(\frac{q_{Rt}}{q_{Pt}} \right)^{\sigma_D-1} \left(\frac{P_P}{P_R} \right)^{\sigma_D}. \quad (21)$$

The first-order conditions for labor and the aggregate digital input imply

$$w_t = (1 - \iota) \frac{Y_{Nt}}{L_{Nt}}, \quad (22)$$

and

$$P_{Dt} = \iota \frac{Y_{Nt}}{\tilde{D}_t} \quad (23)$$

Consequently, expenditure on each digital supplier is

$$P_P m_{Pt} = (1 - s_{Rt}) \iota Y_{Nt}, \quad (24)$$

$$P_R m_{Rt} = s_{Rt} \iota Y_{Nt}. \quad (25)$$

Because the non-digital technology has constant returns to scale, and the sector is competitive, its equilibrium profit is zero:

$$Y_{Nt} - w_t L_{Nt} - P_P m_{Pt} - P_R m_{Rt} = 0 \quad (26)$$

⁵Allowing separate access weights for household and intermediate demand would be a straightforward extension but would require separate empirical discipline.

Improvements in digital quality lower the effective price of digital inputs. Moreover, digital productivity improvements reduce the labor required to serve a given quantity and alter wages, profits, and household income in general equilibrium. Therefore, digital innovation can propagate beyond direct household digital consumption without an additional reduced-form spillover parameter.

2.5 Digital Sector Production

An active digital supplier $j \in \{P, R\}$ produces services using labor:

$$Y_{jt} = A_{jt}L_{jt}, \quad (27)$$

where Y_{jt} is the quantity of digital services output; L_{jt} is labor employed for digital production; $A_{jt} > 0$ is production efficiency. Productivity A_{jt} and consumer-facing service quality q_{jt} are distinct. The former determines the resources required to supply a given quantity, while the latter determines the effective services received by the household based on that quantity.

Digital output is used for both household consumption and non-digital production. Market-clearing for each active supplier requires

$$Y_{jt} = d_{jt} + m_{jt}. \quad (28)$$

It follows that its production labor is

$$L_{jt} = \frac{d_{jt} + m_{jt}}{A_{jt}}. \quad (29)$$

When the rival is inactive,

$$d_{Rt} = m_{Rt} = Y_{Rt} = L_{Rt} = 0. \quad (30)$$

Thus, conditional on posted prices, service quality, and effective market access, active firms accommodate household and intermediate demand.

2.6 Innovation

After observing post-regulation effective market access κ_1 and the entry decision, each active supplier chooses investment in incremental and/or disruptive innovation,

$$x_{jk} \geq 0, \quad j \in \{P, R\} \quad k \in \{I, D\}.$$

Innovation investments are measured in units of the period-0 non-digital numeraire. Their resource cost can be written as

$$\mathcal{C}_j(\mathbf{x}_j) = x_{jI} + x_{jD}. \quad (31)$$

Let $\xi_{jk} \in \{0, 1\}$ indicate whether firm j succeeds in innovation direction k . The probability of success is

$$\theta_{jk}(x_{jk}) \equiv \Pr(\xi_{jk} = 1 \mid x_{jk}) = 1 - \exp(-\eta \zeta_j a_{jk} x_{jk}), \quad \eta > 0, \quad \zeta_j > 0, \quad (32)$$

where η governs the common productivity of innovation expenditure, ζ_j is firm-specific innovation efficiency, and a_{jk} captures comparative advantage across innovation directions. I normalize $\zeta_P = 1$ and allow ζ_R to vary.⁶

⁶The benchmark quantitative application sets $\zeta_R = 1$; alternative values characterize different inherited challenger innovation capacities.

The comparative-advantage parameters satisfy

$$a_{PI} = a_{RD} = \lambda, \quad a_{PD} = a_{RI} = 1 - \lambda, \quad \lambda \in \left(\frac{1}{2}, 1\right).$$

So the platform has a comparative advantage in incremental innovation, while the challenger has a comparative advantage in disruptive innovation, although both firms may invest in both directions. However, the model conditions on these innovation efficiencies at the transition date; it does not identify whether they are intrinsic or partly accumulated under the preceding market regime.

Innovation outcomes are conditionally independent across suppliers and directions. For an active set of firms, the probability of a joint outcome ξ is

$$\Pr(\xi | \mathbf{x}) = \prod_j \prod_{k \in \{I, D\}} \theta_{jk}(x_{jk})^{\xi_{jk}} [1 - \theta_{jk}(x_{jk})]^{1 - \xi_{jk}}. \quad (33)$$

If the rival does not enter, $x_{RI} = x_{RD} = 0$ and its innovation outcomes are excluded from the joint state.

Conditional on successful innovation, technological advancement is

$$\Delta_{j1} = \delta (\xi_{jI} + \gamma_D \xi_{jD}), \quad \delta > 0, \quad \gamma_D > 1. \quad (34)$$

The parameter δ is the size of an incremental technological step, while γ_D measures the size of a disruptive step relative to an incremental step. Quality and production efficiency evolve according to

$$q_{j1} = q_{j0} \exp(\Delta_{j1}), \quad (35)$$

$$A_{j1} = A_{j0} \exp(\Delta_{j1}). \quad (36)$$

Thus, no success leaves (q_{j1}, A_{j1}) at their inherited values; incremental success raises both by $\exp(\delta)$; disruptive success raises both by $\exp(\gamma_D \delta)$; and success in both directions raises both by $\exp[(1 + \gamma_D)\delta]^7$.

2.7 Firms' Optimization Problems

2.7.1 State-Contingent Production

After innovation outcomes are realized, each active supplier chooses output and production labor. In period-1 state ξ , supplier j solves

$$\pi_{j1}^*(\xi) = \max_{Y_{j1}, L_{j1} \geq 0} \{P_j Y_{j1} - w_1 L_{j1}\}, \quad (37)$$

subject to

$$Y_{j1} \leq A_{j1}(\xi) L_{j1}, \quad (38)$$

$$Y_{j1} \leq d_{j1}(\xi; \kappa_1) + m_{j1}(\xi; \kappa_1). \quad (39)$$

Because digital prices are predetermined, I restrict attention to equilibria in which every supplier included in the active choice set is operationally viable:

$$P_j > \frac{w_1(\xi)}{A_{j1}(\xi)}. \quad (40)$$

⁷The equal proportional effect on consumer-facing quality q_{jt} and production efficiency A_{jt} is a baseline restriction.

Under this condition, operating profit increases in output, so the supplier serves all household and intermediate demand. As such, both the production and sales constraints bind.

$$Y_{j1}(\boldsymbol{\xi}) = d_{j1}(\boldsymbol{\xi}; \kappa_1) + m_{j1}(\boldsymbol{\xi}; \kappa_1), \quad (41)$$

$$L_{j1}(\boldsymbol{\xi}) = \frac{d_{j1}(\boldsymbol{\xi}; \kappa_1) + m_{j1}(\boldsymbol{\xi}; \kappa_1)}{A_{j1}(\boldsymbol{\xi})}, \quad (42)$$

$$\pi_{j1}^*(\boldsymbol{\xi}) = \left[P_j - \frac{w_1(\boldsymbol{\xi})}{A_{j1}(\boldsymbol{\xi})} \right] [d_{j1}(\boldsymbol{\xi}; \kappa_1) + m_{j1}(\boldsymbol{\xi}; \kappa_1)]. \quad (43)$$

Demand, wages, and profits are equilibrium objects. Innovation affects profits through two channels: quality changes household and intermediate demand, while productivity changes the labor cost of serving that demand.

2.7.2 The Platform's Problem

Following the entry decision, the platform and the active rival choose innovation simultaneously. The dominant platform chooses innovation before uncertainty is resolved and labor after innovation outcomes are observed. Its problem is

$$\mathcal{V}_P(\kappa_1, e_R) = \max_{x_{PI}, x_{PD} \geq 0} \left\{ -\mathcal{C}_P(\mathbf{x}_P) + \beta \sum_{\boldsymbol{\xi}} \Pr(\boldsymbol{\xi} \mid \mathbf{x}) \pi_{P1}^*(\boldsymbol{\xi}) \right\}, \quad (44)$$

taking the challenger's innovation strategy and the aggregate environment as given. Aggregate wages, income, demand, and profits must subsequently be consistent with the equilibrium allocations described below. Period-0 operating profit is omitted because transition investment does not alter inherited technology and is evaluated conditional on the inherited period-0 operating environment.

2.7.3 The Rival's Entry and Innovation Problem

A previously inactive challenger decides whether to enter before choosing innovation. Conditional on entry, it anticipates the equilibrium platform strategy and state-contingent general-equilibrium profits. Define its optimized continuation value before paying the fixed entry cost as

$$\bar{\mathcal{V}}_R(\kappa_1, \mathcal{H}_0) = \max_{x_{RI}, x_{RD} \geq 0} \left\{ -\mathcal{C}_R(\mathbf{x}_R) + \beta \sum_{\boldsymbol{\xi}} \Pr(\boldsymbol{\xi} \mid \mathbf{x}) \pi_{R1}^*(\boldsymbol{\xi}) \right\}. \quad (45)$$

Its net value of entry is

$$\mathcal{V}_R(\kappa_1, \mathcal{H}_0) = -F_R + \bar{\mathcal{V}}_R(\kappa_1, \mathcal{H}_0), \quad (46)$$

and its entry rule is

$$e_R^*(\kappa_1) = \begin{cases} 1, & \text{if } \mathcal{V}_R(\kappa_1, \mathcal{H}_0) \geq 0, \\ 0, & \text{if } \mathcal{V}_R(\kappa_1, \mathcal{H}_0) < 0. \end{cases} \quad (47)$$

When $\mathcal{V}_R(\kappa, \mathcal{H}_0)$ is continuous and increasing in κ , define the commercial-entry threshold as

$$\kappa^E(\mathcal{H}_0) \equiv \inf \{ \kappa \in [0, 1] : \mathcal{V}_R(\kappa, \mathcal{H}_0) \geq 0 \}. \quad (48)$$

If the threshold lies in the interior of $[0, 1]$, continuity implies

$$\bar{\mathcal{V}}_R(\kappa^E; \mathcal{H}_0) = F_R. \quad (49)$$

If the set in equation (48) is empty, no feasible level of current effective access supports entry from the inherited state. The sufficient conditions for monotonicity and the comparative statics of κ^E are developed in Section 3.

2.7.4 Innovation Probabilities and Expected Profits

For either active supplier and innovation direction $k \in \{I, D\}$, define the state-contingent gain from success as

$$\Delta\pi_{jk}(\boldsymbol{\xi}_{-jk}; \kappa_1) = \pi_{j1}^*(\xi_{jk} = 1, \boldsymbol{\xi}_{-jk}; \kappa_1) - \pi_{j1}^*(\xi_{jk} = 0, \boldsymbol{\xi}_{-jk}; \kappa_1), \quad (50)$$

where $\boldsymbol{\xi}_{-jk}$ collects all innovation outcomes other than ξ_{jk} .

For an interior innovation choice, the first-order condition is

$$1 = \beta\theta'_{jk}(x_{jk})\mathbb{E}_{\boldsymbol{\xi}_{-jk}} [\Delta\pi_{jk}(\boldsymbol{\xi}_{-jk}; \kappa_1)], \quad (51)$$

where

$$\theta'_{jk}(x_{jk}) = \eta\zeta_j a_{jk} \exp(-\eta\zeta_j a_{jk} x_{jk}). \quad (52)$$

The left-hand side is the marginal resource cost of investment. The right-hand side is the marginal increase in the probability of success multiplied by the expected increase in total operating profit generated by that success.

A corner solution $x_{jk} = 0$ arises when

$$\beta\eta\zeta_j a_{jk} \mathbb{E}_{\boldsymbol{\xi}_{-jk}} [\Delta\pi_{jk}(\boldsymbol{\xi}_{-jk}; \kappa_1)] \leq 1. \quad (53)$$

There are no separate “incremental profits” or “disruptive profits”; each innovation direction changes the probability distribution over the firm’s total equilibrium operating profit. Equation (53) provides the basis for the innovation-threshold result derived in Section 3.

2.8 General Equilibrium

The equilibrium combines strategic entry and innovation decisions with state-contingent general-equilibrium allocations. Conditional on an aggregate environment, firms choose entry and innovation. These choices determine innovation probabilities and, after outcomes are realized, qualities, productivities, labor demand, profits, household income, and production. The aggregate variables used in firms’ optimization problems must in turn coincide with those generated by household optimization, firm production, and market clearing.⁸

For a given inherited state $\mathcal{S}_0$ and level of period-1 effective contestability κ_1 , an equilibrium consists of

$$\left\{ e_R^*, \mathbf{x}_P^*, \mathbf{x}_R^*, \mathcal{A}_0^*, \{\mathcal{A}_1^*(\boldsymbol{\xi})\}_{\boldsymbol{\xi} \in \Xi(e_R^*)} \right\},$$

where $\mathcal{A}_0^*$ is the period-0 allocation and $\mathcal{A}_1^*(\boldsymbol{\xi})$ is the period-1 allocation conditional on innovation outcome $\boldsymbol{\xi}$.

These objects satisfy the following conditions.

⁸Thus, equilibrium is a fixed point between strategic behavior and aggregate market conditions. For exposition, I distinguish a strategic block from an aggregate-consistency block. This distinction describes the equilibrium conditions and the numerical solution method; it does not introduce an additional stage into the economic timing.

1. Entry and innovation optimality. Conditional on $e_R^* = 1$, the innovation choices $(\mathbf{x}_P^*, \mathbf{x}_R^*)$ are mutual best responses and solve the platform and rival problems in equations (44) and (45). If $e_R^* = 0$, then

$$\mathbf{x}_R^* = \mathbf{0}, \quad (54)$$

and the platform solves its innovation problem without an active rival.

The entry decision satisfies equation (47). In particular, the rival enters if and only if its discounted expected operating profit covers its fixed entry and innovation costs:

$$\beta \mathbb{E}_0 [\pi_{R1}^*] \geq F_R + \mathcal{C}_R(\mathbf{x}_R^*). \quad (55)$$

2. Innovation outcomes and technological transitions. The probability of each joint innovation outcome is given by equation (33). For every $\boldsymbol{\xi} \in \Xi(e_R^*)$, period-1 quality and productivity satisfy equations (35) and (36).

3. Household optimality. In period 0 and every period-1 innovation state, household consumption satisfies equations (5)–(16). Household income incorporates foreign ownership and the period-0 resource costs of entry and innovation.

4. Firm optimality. In period 0 and every period-1 state, the competitive non-digital firm satisfies equations (20)–(26). Every active digital supplier satisfies the production problem and operational-viability condition in equations (37)–(43). If the rival is inactive, all of its consumption, intermediate-input, production, labor, and profit variables equal zero.

5. Labor market clearing. Labor-market clearing requires

$$L_{N0} + L_{P0} = \bar{L} \quad (56)$$

in period 0 and

$$L_{N1}(\boldsymbol{\xi}) + L_{P1}(\boldsymbol{\xi}) + L_{R1}(\boldsymbol{\xi}) = \bar{L} \quad (57)$$

for every $\boldsymbol{\xi} \in \Xi(e_R^*)$ in period 1.

6. Digital-service market clearing. Digital-service markets are clear according to equation (28) for every active supplier.

7. Non-digital resource feasibility. Foreign owners of the platform receive the fraction ϕ of the dominant platform's net profit. Foreign remittances are

$$T_0^F = \phi [\pi_{P0} - \mathcal{C}_P(\mathbf{x}_P^*)], \quad (58)$$

$$T_1^F(\boldsymbol{\xi}) = \phi \pi_{P1}(\boldsymbol{\xi}). \quad (59)$$

The period-0 non-digital resource constraint is

$$Y_{N0} = C_0 + \mathcal{C}_P(\mathbf{x}_P^*) + e_R^* [F_R + \mathcal{C}_R(\mathbf{x}_R^*)] + T_0^F. \quad (60)$$

For every period-1 innovation state,

$$Y_{N1}(\boldsymbol{\xi}) = C_1(\boldsymbol{\xi}) + T_1^F(\boldsymbol{\xi}). \quad (61)$$

8. Aggregate consistency. The wages, household income, demands, production, and operating profits that firms take as the aggregate environment when evaluating entry and innovation must equal those generated by $\mathcal{A}_0^*$ and $\mathcal{A}_1^*(\boldsymbol{\xi})$. This condition closes the feedback from aggregate conditions to firms' strategies and from firms' collective strategies to the aggregate economy.

2.8.1 Equilibrium Existence and Uniqueness

Appendix A establishes existence under three sufficient conditions. First, every active digital supplier has a strictly positive unit operating surplus in every relevant production state. Second, period-0 entry and innovation expenditures satisfy transition feasibility. Third, each firm's expected innovation payoff is continuous in both firms' strategies and quasi-concave in its own innovation choices over the payoff-relevant strategy set.

Under these conditions, a general equilibrium with endogenous rival entry and strategic innovation exists. Conditional on the market structure, firms' innovation choices, and the realized innovation state, the within-period general-equilibrium allocation is unique. The strategic equilibrium need not be unique because multiplicity may arise from the discrete entry decision or from the strategic innovation subgame. Appendix A provides the formal result and proof.

3 Economic Mechanisms and Model Implications

This section derives the economic implications of the model before imposing a particular calibration. The model delivers two kinds of results. First, the demand system yields direct implications for rival commercialization and quality-adjusted digital prices, conditional on entry and a given price-quality state. Second, the strategic model generates thresholds for commercially viable entry and challenger innovation under transparent regularity conditions on equilibrium values and marginal returns. Separating these results clarifies which conclusions follow directly from the model's structure and which depend on the inherited competitive state and the quantitative discipline imposed later.

Throughout this section, κ denotes period-1 effective market access, with the time subscript suppressed. The inherited non-access conditions are summarized by $\mathcal{H}_0$. Because the continuation innovation game may admit multiple equilibria, the threshold results either assume that the continuation equilibrium is unique or refer to a fixed continuous equilibrium selection. All value functions and marginal returns below are understood with respect to that selection.

The results are conditional on $\mathcal{H}_0$. They do not assume that the inherited state developed independently of earlier market-access conditions. In particular, inherited differences in quality, productivity, networks, data, reputation, or innovation capacity may partly reflect the legacy of the earlier market regime.

3.1 Effective Market Access and Rival Commercialization

Consider a period-1 innovation state $\boldsymbol{\xi}$ in which the challenger is active. Define the state-contingent relative-demand term

$$\Omega(\boldsymbol{\xi}) \equiv \left(\frac{p_R(\boldsymbol{\xi})}{p_P(\boldsymbol{\xi})} \right)^{1-\sigma_D} > 0. \quad (62)$$

Using equation (9), the challenger's share of total digital expenditure can be written as

$$s_R(\kappa; \xi) = \frac{\kappa\Omega(\xi)}{1 + \kappa\Omega(\xi)}. \quad (63)$$

The term $\Omega(\xi)$ summarizes the challenger's relative price-quality position in the realized state. It is held fixed in the direct comparative statics below.

Proposition 1 (Effective market access and commercialization). *Fix an innovation state ξ and suppose that the challenger is active. Holding the suppliers' quality-adjusted prices fixed, the challenger's digital-expenditure share is increasing and concave in effective market access:*

$$\frac{\partial s_R}{\partial \kappa} = \frac{\Omega}{(1 + \kappa\Omega)^2} > 0, \quad (64)$$

$$\frac{\partial^2 s_R}{\partial \kappa^2} = -\frac{2\Omega^2}{(1 + \kappa\Omega)^3} < 0. \quad (65)$$

For $\kappa > 0$, the elasticity of the challenger's expenditure share with respect to effective market access is

$$\frac{\partial \log s_R}{\partial \log \kappa} = 1 - s_R, \quad (66)$$

and the challenger's expenditure-share odds satisfy

$$\frac{s_R}{1 - s_R} = \kappa\Omega. \quad (67)$$

At the full market access, $\kappa = 1$,

$$s_R^{FA}(\xi) = \frac{\Omega(\xi)}{1 + \Omega(\xi)}. \quad (68)$$

Thus, full access does not generally imply equal market shares.

Proof. The results follow by differentiating equation (63). Equation (67) follows by rearrangement, and equation (68) follows by setting $\kappa = 1$. $\square$

[Symmetric price-quality benchmark] If the platform and challenger have the same quality-adjusted price in a given state, so that $p_R = p_P$ and $\Omega = 1$, then

$$s_R(\kappa) = \frac{\kappa}{1 + \kappa}. \quad (69)$$

The full-access benchmark then produces a fifty-fifty expenditure split. If the rival has a higher quality-adjusted price than the platform, its full market access share is below one-half; if it has a lower quality-adjusted price, its full-access share is above one-half. Figure 1 illustrates this idea.

Proposition 1 distinguishes removal of the modeled access disadvantage from removal of all competitive asymmetries. The benchmark $\kappa = 1$ equalizes the firms' market-access weights. It does not equalize prices, qualities, productivities, networks, data, reputation, or other components of the inherited competitive state. Nor does it impose homogeneous-product Bertrand competition.

The share-odds relationship also gives a useful accounting decomposition. For an observed pair $(\bar{\kappa}, \bar{s}_R)$ with $\bar{\kappa} > 0$, the model-implied relative-demand term is

$$\bar{\Omega} = \frac{\bar{s}_R}{\bar{\kappa}(1 - \bar{s}_R)}. \quad (70)$$

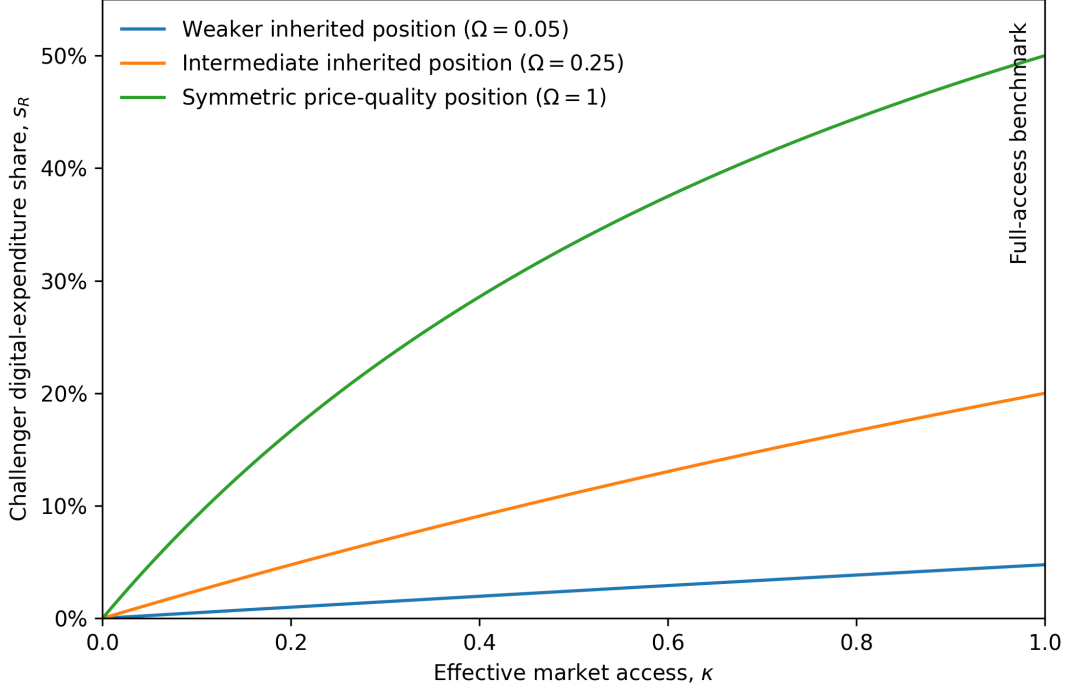


Figure 1: Market Share of the Rivals

Holding that price–quality state fixed, the direct commercialization effect of moving to an alternative access level κ' is

$$s_R^{\text{dir}}(\kappa'; \bar{\kappa}, \bar{s}_R) = \frac{\kappa' \bar{s}_R}{\bar{\kappa}(1 - \bar{s}_R) + \kappa' \bar{s}_R}. \quad (71)$$

This is a within-model accounting relationship, not a causal decomposition of the challenger’s observed market position. The residual term $\bar{\Omega}$ may reflect contemporaneous product or preference differences, but it may also reflect scale, network, data, reputation, and product-development gaps accumulated under the preceding market regime. The model does not identify those historical components separately. In the full equilibrium, innovation responses may also change relative quality-adjusted prices, so equation (71) isolates the direct market access channel from the comprehensive response.

3.2 Commercial Viability and the Entry Threshold

Proposition 1 shows that, conditional on rival activity and a fixed price–quality state, greater effective market access increases the share of digital expenditure allocated to the rival. This direct commercialization effect does not imply that the rival will enter. Entry requires the value of the commercial opportunity created by market access to cover both the fixed cost of entry and any optimally chosen innovation expenditure.

Let

$$\mathcal{V}_R^E(\kappa; \mathcal{H}_0) \equiv \bar{\mathcal{V}}_R(\kappa; \mathcal{H}_0) - F_R \quad (72)$$

denote the challenger’s equilibrium net value of entry. The continuation value $\bar{\mathcal{V}}_R(\kappa; \mathcal{H}_0)$ incorporates the challenger’s optimal innovation choices and its expected state-contingent general-equilibrium operating profits conditional on entry.

Because the continuation innovation game may admit multiple equilibria, I fix an equilibrium selection when evaluating $\mathcal{V}_R^E(\kappa; \mathcal{H}_0)$.

Single-threshold entry-support condition. Define the set of effective-market-access states under which entry is privately viable:

$$\mathcal{K}^E(\mathcal{H}_0) \equiv \{\kappa \in [\kappa_0, 1] : \mathcal{V}_R^E(\kappa; \mathcal{H}_0) \geq 0\}. \quad (73)$$

Suppose that $\mathcal{V}_R^E(\kappa; \mathcal{H}_0)$ is continuous and that $\mathcal{K}^E(\mathcal{H}_0)$ is either empty or an upper interval of $[\kappa_0, 1]$. Equivalently, once entry becomes commercially viable, it remains viable at every higher level of effective market access.⁹

The direct demand effect established in Proposition 1 provides one force supporting this property: holding the price-quality state fixed, greater effective market access increases the challenger's expenditure share. It does not establish the single-threshold condition by itself. In the full model, effective market access also changes total digital expenditure, equilibrium wages, operating margins, and firms' innovation choices. The entry-support condition is restriction on the overall equilibrium response. It holds analytically in the transparent benchmark below and is evaluated numerically in the quantitative application.

Proposition 2 (Commercial-entry threshold). *Suppose that the net entry value is continuous and satisfies the single-threshold entry-support condition. Suppose also that the inherited market is consistent with non-entry:*

$$\mathcal{V}_R^E(\kappa_0; \mathcal{H}_0) < 0. \quad (74)$$

Then exactly one of the following cases applies:

1. **Entry is not viable at any feasible access level.**

If

$$\mathcal{V}_R^E(1; \mathcal{H}_0) < 0, \quad (75)$$

then

$$\mathcal{K}^E(\mathcal{H}_0) = \emptyset.$$

Even the full market access benchmark cannot generate sufficient commercial value to cover the fixed entry cost from the inherited state.

2. **A unique commercial-entry threshold exists.**

If

$$\mathcal{V}_R^E(1; \mathcal{H}_0) \geq 0, \quad (76)$$

then there exists a unique threshold $\kappa^E \in (\kappa_0, 1]$ such that

$$\mathcal{K}^E(\mathcal{H}_0) = [\kappa^E, 1],$$

and

$$\mathcal{V}_R^E(\kappa^E; \mathcal{H}_0) = 0. \quad (77)$$

Under the convention that the challenger enters when indifferent, its equilibrium entry decision is

$$e_R^*(\kappa) = \begin{cases} 0, & \kappa < \kappa^E, \\ 1, & \kappa \geq \kappa^E. \end{cases} \quad (78)$$

⁹This condition is weaker than requiring the net entry value to be strictly increasing over the entire access range. The entry value may flatten or decline over some range after entry becomes viable, provided that it does not fall below zero again. The condition is sufficient to represent the entry decision using a single cutoff.

The minimum improvement in realized effective market access required to induce entry from the inherited state is

$$\Delta\kappa^E = \kappa^E - \kappa_0. \quad (79)$$

Proof. If $\mathcal{V}_R^E(1; \mathcal{H}_0) < 0$, the entry-support set cannot be nonempty. If it contained some $\kappa < 1$, the upper-set condition would imply that $1 \in \mathcal{K}^E(\mathcal{H}_0)$, contradicting equation (75). If $\mathcal{V}_R^E(1; \mathcal{H}_0) \geq 0$, then $1 \in \mathcal{K}^E(\mathcal{H}_0)$, so the entry-support set is nonempty.

Define $\kappa^E \equiv \inf \mathcal{K}^E(\mathcal{H}_0)$. Continuity of the net entry value implies that $\mathcal{K}^E(\mathcal{H}_0)$ is closed, so $\kappa^E \in \mathcal{K}^E(\mathcal{H}_0)$. The inherited non-entry condition and continuity imply that $\kappa^E > \kappa_0$. The upper-set condition then gives $\mathcal{K}^E(\mathcal{H}_0) = [\kappa^E, 1]$. For every $\kappa < \kappa^E$, the net entry value is negative, whereas it is nonnegative at κ^E . Continuity implies $\mathcal{V}_R^E(\kappa^E; \mathcal{H}_0) = 0$. The entry rule follows from the definition of the entry-support set and the entry-at-indifference convention. $\square$

Proposition 2 connects the direct commercialization mechanism to the rival's entry decision. Proposition 1 describes how market access affects the rival's expenditure share *conditional on being active*. Proposition 2 determines whether the commercial opportunity is valuable enough for the rival to become active in the first place.

Below κ^E , improved effective market access may increase the sales that the rival could obtain if it entered, but that latent commercial opportunity remains insufficient to cover the fixed entry cost. The challenger, therefore, remains outside the market, and its realized expenditure share is zero. At κ^E , entry becomes privately viable. The challenger's observed market presence consequently changes discontinuously at the threshold, even though its expenditure share varies smoothly with κ conditional on entry. Above κ^E , further improvements in effective market access continue to expand the commercial scale of an active challenger.

The proposition also shows why the magnitude of a market-access improvement cannot be evaluated independently of the market's starting position. A large increase from a very low κ_0 may leave the market below κ^E , while a small increase from just below κ^E may induce entry. What matters is whether the realized post-intervention state crosses the threshold, not merely the size of the change in isolation.

The threshold is itself conditional on the inherited competitive conditions $\mathcal{H}_0$ and the model's structural primitives. Fixed entry costs, market size, the challenger's inherited relative-demand position, productivity, and innovation opportunities all affect the value of entry. These inherited conditions should not be interpreted as necessarily intrinsic to the challenger or independent of previous contestability. They may partly reflect scale, network, data, reputation, and product-development opportunities accumulated under the preceding market regime.

The first case in Proposition 2 is economically substantive. Removing the entire contemporaneous market-access disadvantage represented by κ need not induce entry if the continuation value generated from the inherited state remains below the fixed entry cost. Full market access and commercially viable entry are distinct concepts.

Finally, $\Delta\kappa^E$ measures the required improvement in the *realized effective-market-access state*, conditional on the inherited competitive state. It does not measure the legal strictness, enforcement expenditure, or number of regulatory obligations required to produce that improvement. The model does not specify a policy technology mapping legal or enforcement choices into κ .

[Local comparative statics of the entry threshold] Suppose that $\kappa^E \in (\kappa_0, 1)$ is an interior threshold, that the net entry value is differentiable in a neighborhood of κ^E , and that the threshold is a regular crossing:

$$\left. \frac{\partial \mathcal{V}_R^E}{\partial \kappa} \right|_{\kappa=\kappa^E} > 0. \quad (80)$$

Then the threshold is increasing in the fixed entry cost:

$$\frac{\partial \kappa^E}{\partial F_R} = \frac{1}{\partial \bar{\mathcal{V}}_R / \partial \kappa} > 0. \quad (81)$$

For any inherited condition or structural parameter h that raises the challenger's continuation value locally,

$$\frac{\partial \bar{\mathcal{V}}_R}{\partial h} > 0, \quad (82)$$

the threshold satisfies

$$\frac{\partial \kappa^E}{\partial h} = -\frac{\partial \bar{\mathcal{V}}_R / \partial h}{\partial \bar{\mathcal{V}}_R / \partial \kappa} < 0. \quad (83)$$

Proof. The results follow from the implicit-function theorem applied to

$$\bar{\mathcal{V}}_R(\kappa^E; \mathcal{H}_0) - F_R = 0. \quad (84)$$

□

The corollary gives local comparative statics around a threshold. It does not assert that the equilibrium entry value is globally monotone under every parameterization. A higher fixed cost raises the amount of effective market access required for entry. Conversely, any inherited condition or structural parameter that raises the challenger's continuation value lowers the required access threshold. The economic source of that change must be interpreted through the relevant parameter rather than treated as a generic measure of challenger merit or effort.

A transparent benchmark. The threshold mechanism and the entry-support condition can be seen directly in a simplified environment. Hold innovation choices, total digital expenditure, and the challenger's operating profit share fixed. Let $E^D > 0$ denote total digital expenditure and let

$$\nu_R \equiv 1 - \frac{w}{P_R A_R} > 0 \quad (85)$$

be the challenger's operating-profit share of revenue. The net value of entry is then

$$\mathcal{V}_R^{E, \text{bench}}(\kappa) = \beta \nu_R E^D \frac{\kappa \Omega}{1 + \kappa \Omega} - F_R. \quad (86)$$

In this benchmark,

$$\frac{\partial \mathcal{V}_R^{E, \text{bench}}}{\partial \kappa} = \beta \nu_R E^D \frac{\Omega}{(1 + \kappa \Omega)^2} > 0. \quad (87)$$

The entry value is therefore strictly increasing, and the entry-support set necessarily has the upper-interval structure imposed above.

Entry is viable for some $\kappa \leq 1$ if and only if

$$F_R \leq \beta \nu_R E^D \frac{\Omega}{1 + \Omega}. \quad (88)$$

When this condition holds, the benchmark threshold is

$$\kappa^{E, \text{bench}} = \frac{F_R}{\Omega (\beta \nu_R E^D - F_R)}. \quad (89)$$

The closed-form expression makes the mechanism transparent. The threshold rises with the fixed entry cost and falls with total market expenditure, the challenger's operating profitability, and its inherited relative-demand position. If equation (88) fails, even the full market access benchmark does not create a sufficiently valuable commercial base to cover the fixed entry cost in the simplified environment.

3.3 Commercial Scale and the Rival-Innovation Threshold

Entry does not imply that the rival has enough commercial scale to invest in innovation. For innovation direction $k \in \{I, D\}$, define

$$\chi_{Rk} \equiv \theta'_{Rk}(0) > 0 \quad (90)$$

as the rival's marginal innovation productivity at zero investment.¹⁰

Let

$$\mathcal{M}_{Rk}(\kappa; \mathcal{H}_0) \equiv \mathbb{E}_{\xi_{-Rk}} [\Delta\pi_{Rk}(\xi_{-Rk}; \kappa)] \quad (91)$$

be the rival's expected increase in period-1 operating profit generated by successful innovation in direction k , evaluated at $x_{Rk} = 0$ and along the selected continuation equilibrium for the remaining innovation choices. The net marginal return to initiating innovation is

$$\mathcal{G}_{Rk}(\kappa; \mathcal{H}_0) \equiv \beta\chi_{Rk}\mathcal{M}_{Rk}(\kappa; \mathcal{H}_0) - 1. \quad (92)$$

Under concavity of the challenger's payoff in x_{Rk} , the optimal investment is zero when $\mathcal{G}_{Rk} \leq 0$ and positive when $\mathcal{G}_{Rk} > 0$.

Scale-return. Suppose that, conditional on entry, $\mathcal{M}_{Rk}(\kappa; \mathcal{H}_0)$ is continuous and strictly increasing in effective market access κ . Economically, this condition requires the expansion of the commercial base to increase the profit gain generated by a successful innovation after accounting for equilibrium wage, demand, and strategic responses.

Proposition 3 (Rival innovation threshold and access-capacity complementarity). *Suppose scale-return holds for innovation direction k .*

1. *If $\mathcal{G}_{Rk}(\kappa^E) > 0$, innovation in direction k becomes profitable as soon as the rival enters. In this case, set $\kappa_k^I = \kappa^E$.*
2. *If $\mathcal{G}_{Rk}(\kappa^E) \leq 0 < \mathcal{G}_{Rk}(1)$, there exists a unique innovation threshold $\kappa_k^I \in [\kappa^E, 1)$ satisfying*

$$\beta\chi_{Rk}\mathcal{M}_{Rk}(\kappa_k^I; \mathcal{H}_0) = 1. \quad (93)$$

The rival chooses $x_{Rk}^* = 0$ below the threshold and $x_{Rk}^* > 0$ above it.

3. *If $\mathcal{G}_{Rk}(1) \leq 0$, no feasible level of effective market access makes innovation in direction k privately profitable from the inherited state. In this case, write $\kappa_k^I = +\infty$.*

For an interior threshold, greater marginal innovation capacity lowers the effective market access required to activate innovation:

$$\frac{\partial \kappa_k^I}{\partial \chi_{Rk}} = -\frac{\mathcal{M}_{Rk}(\kappa_k^I)}{\chi_{Rk}\mathcal{M}'_{Rk}(\kappa_k^I)} < 0. \quad (94)$$

Moreover, the marginal return to innovation exhibits complementarity between effective market access and innovation capacity:

$$\frac{\partial^2}{\partial \kappa \partial \chi_{Rk}} [\beta\chi_{Rk}\mathcal{M}_{Rk}(\kappa)] = \beta\mathcal{M}'_{Rk}(\kappa) > 0. \quad (95)$$

Proof. Under scale-return assumption, $\mathcal{G}_{Rk}$ is continuous and strictly increasing in κ . The three cases follow from its sign at κ^E and at 1, together with the innovation corner condition. Equation (94) follows by differentiating equation (93). Equation (95) follows directly from the definition of the marginal return. $\square$

¹⁰In the baseline success technology, $\chi_{Rk} = \eta a_{Rk}$; if a challenger-specific efficiency shifter is included, $\chi_{Rk} = \eta \zeta_{Rk} a_{Rk}$.

Three rival-entry and innovation regimes. Let

$$\kappa^I \equiv \min_{k \in \{I, D\}} \kappa_k^I$$

denote the first effective market access level at which at least one direction of rival innovation becomes privately viable. When

$$\kappa^E < \kappa^I \leq 1,$$

the model divides the effective-market-access interval into three regimes:

Regime I: No rival entry	$0 \leq \kappa < \kappa^E,$
Regime II: Rival entry without rival innovation	$\kappa^E \leq \kappa < \kappa^I,$
Regime III: Rival entry with rival innovation	$\kappa^I \leq \kappa \leq 1.$

In Regime I, the commercial opportunity available to the rival is insufficient to cover the fixed entry cost. In Regime II, entry is commercially viable, but the resulting sales base is not large enough to make rival innovation privately profitable. In Regime III, the rival both enters and invests in at least one direction of innovation.

More specifically, at $\kappa = \kappa^E$, the maintained tie-breaking rule selects entry. At an interior κ^I satisfying a zero net marginal return, the rival remains at the innovation corner and invests strictly positively immediately above the threshold. If $\kappa^I = \kappa^E$, entry and innovation are activated together. If $\kappa^I = +\infty$, removing the entire modeled market-access disadvantage is insufficient to activate rival innovation from the inherited state.

These results make the effect of an access improvement state-dependent. The same change $\Delta\kappa$ may leave the market below the entry threshold, cross the entry threshold without crossing the innovation threshold, or activate challenger innovation. The relevant outcome depends on the market's initial distance from the two thresholds, not only on the absolute size of the access change.

The parameter χ_{Rk} and the other components of $\mathcal{H}_0$ describe capacities inherited at the transition. Proposition 3 does not imply that these capacities are intrinsic to the challenger or independent of earlier market closure. Restricted historical access may itself have limited scale, learning, data accumulation, and product development. The proposition establishes how current access and inherited capacity interact, conditional on that history.

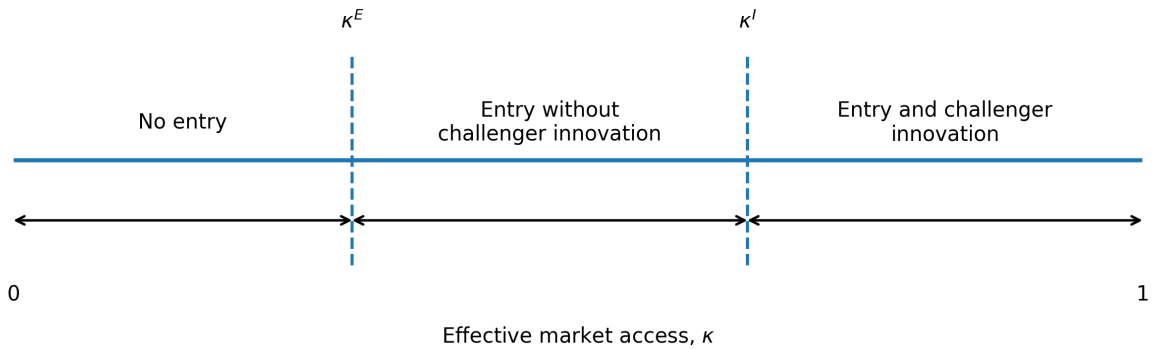


Figure 2: Different Regimes of the Model

3.4 Platform Innovation: Business Stealing and Market Expansion

Greater effective market access directly shifts expenditure toward the rival but it may also change the size of the digital market. Let total digital expenditure in a given state be

$$E^D(\kappa) \equiv S^H(\kappa)I(\kappa) + \iota Y_N(\kappa). \quad (96)$$

Because households and the non-digital producer use the same within-digital aggregator, platform and challenger revenues are

$$\mathcal{R}_P(\kappa) = [1 - s_R(\kappa)] E^D(\kappa), \quad \mathcal{R}_R(\kappa) = s_R(\kappa) E^D(\kappa). \quad (97)$$

Holding the realized relative price–quality state fixed, Proposition 1 implies

$$\frac{\partial \log[1 - s_R]}{\partial \log \kappa} = -s_R, \quad (98)$$

$$\frac{\partial \log s_R}{\partial \log \kappa} = 1 - s_R. \quad (99)$$

Therefore,

$$\frac{\partial \log \mathcal{R}_P}{\partial \log \kappa} = \underbrace{\frac{\partial \log E^D}{\partial \log \kappa}}_{\text{market-size effect}} - \underbrace{s_R}_{\text{business-stealing effect}}, \quad (100)$$

$$\frac{\partial \log \mathcal{R}_R}{\partial \log \kappa} = \frac{\partial \log E^D}{\partial \log \kappa} + (1 - s_R). \quad (101)$$

The sign of the platform’s revenue response depends on whether the equilibrium market-size effect offsets the direct loss of expenditure share.

For innovation direction k , let

$$\mathcal{M}_{Pk}(\kappa) \equiv \mathbb{E}_{\xi_{-Pk}} [\Delta \pi_{Pk}(\xi_{-Pk}; \kappa)] \quad (102)$$

be the platform’s expected operating-profit gain from a successful innovation. Let $\chi_{Pk} = \theta'_{Pk}(0)$. Conditional on the opponent’s strategy and the platform’s other innovation choice, the exponential success technology implies the best response

$$x_{Pk}^*(\kappa) = \max \left\{ 0, \frac{1}{\chi_{Pk}} \log [\beta \chi_{Pk} \mathcal{M}_{Pk}(\kappa)] \right\}. \quad (103)$$

Proposition 4 (Ambiguous platform-innovation response). *Conditional on an interior best response, the platform’s innovation in direction k increases with effective market access if and only if effective market access raises the expected profit gain from a successful innovation:*

$$\text{sign} \left(\frac{\partial x_{Pk}^*}{\partial \kappa} \right) = \text{sign} \left(\frac{\partial \mathcal{M}_{Pk}}{\partial \kappa} \right). \quad (104)$$

The model does not impose an unconditional sign on this derivative. Greater effective market access weakens the platform’s appropriability through business stealing, but it may expand aggregate digital demand, change production costs and wages, and alter the challenger’s innovation strategy. The platform’s equilibrium innovation may therefore rise or fall with effective market access.

Proof. When the best response is interior, differentiating equation (103) gives

$$\frac{\partial x_{P_k}^*}{\partial \kappa} = \frac{1}{\chi_{P_k} \mathcal{M}_{P_k}(\kappa)} \frac{\partial \mathcal{M}_{P_k}(\kappa)}{\partial \kappa}. \quad (105)$$

The remaining statement follows because the components of $\mathcal{M}_{P_k}$ need not move in the same direction as κ . $\square$

Platform profit and platform innovation need not move together. Profit is a level outcome, whereas innovation depends on the profit difference between success and failure. Greater market access can reduce the platform's total profit while increasing the incremental value of improving quality or productivity over a larger digital market. The quantitative application determines which force dominates in the calibrated market.

3.5 Digital Prices, Downstream Propagation, and Welfare

The demand system produces a direct price-index effect even when the rival remains small. Conditional on challenger activity and a fixed price-quality state,

$$P_D(\kappa) = \left[p_P^{1-\sigma_D} + \kappa p_R^{1-\sigma_D} \right]^{\frac{1}{1-\sigma_D}}. \quad (106)$$

Proposition 5 (Direct price and downstream-propagation effects). *Fix an innovation state in which the challenger is active and hold quality-adjusted supplier prices fixed. For $\kappa > 0$:*

1. *Effective market access lowers the quality-adjusted digital price index:*

$$\frac{\partial \log P_D}{\partial \log \kappa} = -\frac{s_R}{\sigma_D - 1} < 0. \quad (107)$$

2. *Effective market access lowers the household consumption price index:*

$$\frac{\partial \log P_X}{\partial \log \kappa} = -S^H \frac{s_R}{\sigma_D - 1} < 0. \quad (108)$$

3. *If the competitive non-digital sector is active, its unit-cost condition implies*

$$w = (1 - \iota) B^{\frac{1}{1-\iota}} \left(\frac{\iota}{P_D} \right)^{\frac{\iota}{1-\iota}}, \quad (109)$$

and hence

$$\frac{\partial \log w}{\partial \log \kappa} = \frac{\iota}{1 - \iota} \frac{s_R}{\sigma_D - 1} > 0. \quad (110)$$

Proof. Differentiating equation (106) gives equation (107). From the upper-tier CES price index, $\partial \log P_X / \partial \log P_D = S^H$, which yields equation (108). Equation (109) follows from the non-digital producer's zero-profit unit-cost condition; combining it with equation (107) gives equation (110). $\square$

Holding household income fixed, the direct effect on real consumption is

$$\left. \frac{\partial \log X}{\partial \log \kappa} \right|_I = S^H \frac{s_R}{\sigma_D - 1} > 0. \quad (111)$$

The wage response in Proposition 5 shows how a change in the digital bottleneck can propagate beyond direct household digital spending: lower quality-adjusted digital-input costs raise the marginal product of labor in non-digital production.

The proposition does not establish that total expected welfare must increase for every parameterization. Effective market access also changes platform and challenger profits, foreign remittances, entry and innovation expenditures, endogenous innovation probabilities, and the allocation of labor. Period-0 entry and innovation costs can offset part of the period-1 price and productivity gains. The sign and magnitude of the complete welfare effect are therefore quantitative general-equilibrium questions.

3.6 Summary of the Theoretical Mechanism

The model separates five steps that are often treated as a single competition response. First, effective market access expands the demand that a rival can commercialize conditional on entry. Second, entry occurs only when the value of that commercial opportunity crosses the fixed-cost threshold κ^E . Third, rival innovation requires a further scale-adjusted return and may therefore begin only at a second threshold κ^I . Fourth, the platform’s innovation response is ambiguous because business stealing and market expansion affect its marginal innovation return in opposite directions. Fifth, changes in the quality-adjusted digital price propagate to the broader economy through household consumption and the use of digital services in non-digital production.

These implications are conditional on the inherited competitive state. A persistent improvement in market access could, over a longer horizon, alter that state by changing user scale, network formation, data accumulation, reputation, and repeated innovation. The two-period model does not specify that recursive process. Its purpose is instead to characterize the transition from an inherited platform-only market and to identify the market-access conditions under which entry and one round of challenger innovation become privately viable.

The quantitative application in the following section uses these theoretical objects in three ways. It measures the inherited and post-intervention access states, locates them relative to κ^E and κ^I , and quantifies the general-equilibrium commercialization, innovation, and welfare effects that cannot be signed from the analytical results alone.

4 Quantitative Application: Mobile App Distribution in the European Union

The theoretical analysis leaves three questions to the quantitative application. First, where do the inherited and observed market-access states lie relative to the commercial-entry and rival-innovation thresholds? Second, how much commercialization and innovation result from moving across alternative effective-market-access states? Third, how do those market-level changes propagate through digital prices, production, income, profits, and household welfare?

I study these questions in mobile app distribution on Apple devices in the European Union. The application does not treat the DMA as a policy concerned only with market access. The DMA pursues the broader objectives of contestability and fairness through multiple obligations. App distribution is useful here because one important part of that broader regime—the opening and practical usability of alternative distribution channels—maps particularly closely into the market-access mechanism isolated by the model.

4.1 Why Mobile App Distribution?

Before the implementation of the DMA, Apple’s App Store was the only generally available channel through which developers could distribute native applications to those users of Apple

devices, apart from limited enterprise and testing exceptions. This narrowly defined inherited market maps naturally into the model’s maintained initial condition: the platform is active, while a potential challenger is not yet commercially active.

The platform in the model corresponds to the incumbent app-distribution channel. The rival represents a potential alternative marketplace or distribution provider. Because several alternative marketplaces entered after the opening, the modeled challenger should be interpreted as a representative alternative distribution channel rather than as a literal description of one named firm. Effective market access captures the practical ability of such a channel to reach iOS users and convert potential interest into completed adoption and expenditure.

The application is also useful because app distribution is an upstream gateway to economic activity beyond direct purchases from an app store. Burley and Fradkin (2025) estimate that physical goods and services accounted for \$1.014 trillion, or 78 percent, of the billings and sales facilitated by the global App Store ecosystem in 2024. For Europe, they estimate \$109 billion in physical goods and services transactions out of \$148 billion in total facilitated billings and sales. These estimates are industry-supported and are used only as descriptive evidence that app-mediated services connect to activity outside the narrowly measured digital sector. They do not discipline the model’s digital-input share.

Finally, mobile app distribution provides comparatively rich information on the implementation of access obligations, alternative marketplace entry, user conversion, platform financial performance, research and development, and market outcomes. Some important objects remain imperfectly observed—especially market-specific R&D and the strength of downstream input linkages—so the exercise is a disciplined reference calibration rather than a statistically estimated structural model.

4.2 From Formal Opening to Usable Market Access

Article 6(4) of the DMA requires designated gatekeepers to permit the installation and effective use of third-party applications and app stores (European Parliament and Council of the European Union, 2022). Apple subsequently introduced the technical frameworks required for alternative iOS distribution in the European Union (Apple, 2024). AltStore PAL, Aptoide, the Epic Games Store, and Setapp Mobile entered following this opening. The change had a real effect: distribution services that could not previously reach ordinary iOS users became operational.

Formal availability did not place alternative channels on commercially equivalent terms. In April 2025, the European Commission reached the preliminary view that Apple’s business terms, eligibility conditions, and installation procedures discouraged the use of alternative distribution channels (European Commission, 2025). Epic Games reports that, after Apple shortened the installation procedure for an alternative marketplace from fifteen steps to six; the share of interested users abandoning an attempted installation declined from approximately 65 percent to 25 percent (Epic Games, 2025). The implied completion rate among users who had already initiated installation increased from approximately 35 percent to 75 percent.

The Epic comparison is reported by an interested market participant and is not a controlled estimate of the DMA’s aggregate effect. It nevertheless illustrates the behavior represented by effective market access in the model: conditional on potential interest in a rival, practical market access conditions determine how much of that demand can be realized.

Operational entry also did not guarantee continued commercial viability. Setapp Mobile entered with a differentiated subscription-based marketplace but ceased operating in February

2026. MacPaw attributed the decision to evolving and complex business terms that no longer fit its business model (MacPaw, 2024, 2026). The combination of new entry, continuing adoption frictions, and subsequent exit by one entrant motivates the model’s distinction among formal opening, usable market access, commercially viable entry, and sustained innovation.

4.3 Measuring Effective Market Access

The model uses κ_t as a reduced-form effective-market-access weight. To discipline this object, the application draws on the companion effective-contestability framework in Cheng, Xu, and Yakout (2026). That framework is broader than the single channel represented in the model. It evaluates whether competitive opportunities are formally established and whether they can be used in practice across four dimensions: user mobility and switching; challenger access and distribution; platform conduct and neutrality; and data, interoperability, and ecosystem access.

Let $\hat{\kappa}_t^J$ and $\hat{\kappa}_t^F$ denote the companion framework’s de jure and de facto scores. Its aggregate effective-contestability score is

$$\hat{\kappa}_t^C = \hat{\kappa}_t^J \hat{\kappa}_t^F. \quad (112)$$

The multiplicative form treats formal coverage and practical usability as complements. The full methodology, indicator definitions, source documentation, and validation exercises remain in Appendix C and the companion paper.

For the quantitative application, I normalize the model’s effective market-access weight to equal the measured effective-contestability score:

$$\kappa_t \equiv \hat{\kappa}_t^C. \quad (113)$$

This is a reduced-form empirical mapping, not a claim that every component of effective contestability operates only through contemporaneous user access. Some measured conditions may also affect accumulated networks, data, reputation, complementary investment, or innovation capacity. The baseline model conditions on those inherited effects through $\mathcal{H}_0$ rather than deriving their history.

The index excludes entry, expenditure shares, revenue, profits, and innovation because these are equilibrium outcomes the model seeks to explain. For example, an installation procedure may inform the practical usability score, but the resulting installation or expenditure share does not.

For EU app distribution on iOS, the inherited de jure and de facto scores are 0.18 and 0.08:

$$\kappa_0 = 0.18 \times 0.08 = 0.0144.$$

After DMA implementation, the formal obligations cover the access channels included in the measure, producing a de jure score of one, while the measured de facto score is 0.47:

$$\kappa_1^{\text{obs}} = 1.00 \times 0.47 = 0.47.$$

The post-DMA implementation de jure score of one means that the formal obligations cover the channels included in the index. It does not mean that every aspect of app-distribution competition is perfectly regulated. The de facto score records the substantial practical frictions that remain.

The two measured values describe inherited and observed market states. They are not direct policy choices. The counterfactuals below vary κ_1 to compare equilibrium outcomes across alternative effective market access environments; they do not estimate a response function from particular legal obligations or enforcement expenditures to κ_1 .

Table 1: **Effective-contestability scores and their market-access mapping**

Market state	$\hat{\kappa}_t^J$	$\hat{\kappa}_t^F$	Model access weight κ_t
Inherited market	0.18	0.08	0.0144
Observed post-implementation market	1.00	0.47	0.4700

Notes: The first two columns come from the companion effective-contestability framework. Equation (113) maps their product into the model’s effective-market-access weight. The mapping is a normalization for the quantitative exercise.

5 Calibration and Counterfactual Design

The theoretical results raise three quantitative questions. First, has the observed post-DMA improvement in effective market access moved the market beyond the threshold for commercially viable entry, and how close is the market to the higher threshold for rival innovation? Second, if the remaining contemporaneous market access disadvantage were removed while challengers retained the competitive position inherited at the time of reform, how much further would they scale? Third, how would entry, innovation, market shares, and welfare change if those inherited competitive gaps were narrower?

These questions matter because a small post-reform rival share does not have a single interpretation. It may reflect remaining limits on current market access, but it may also reflect inherited differences in quality, productivity, network reach, data, reputation, complementary services, and prior opportunities to develop and scale. The model represents current market access and inherited competitive conditions as separate objects, but it does not causally identify the historical source of the inherited position.

The quantitative analysis begins by locating the observed market relative to the entry and innovation thresholds. It then examines separately how outcomes change when current market access improves and when the challenger’s inherited competitive position becomes stronger.

5.1 Periodization and Calibration Strategy

Period 0 is the pre-regulation equilibrium in 2023, and period 1 represents the post-regulation market in 2025. I set the annual discount factor to 0.96, which implies

$$\beta = 0.96^2 = 0.9216.$$

The calibration combines normalizations, empirically matched moments, and assigned parameters. Scale variables are normalized. Parameters with observable counterparts are chosen to match those moments. Parameters that cannot be identified from app-distribution data—especially substitution, the downstream digital-input share, inherited productivity differences, and the composition of innovation—are assigned benchmark values and varied in sensitivity analysis.

The resulting calibration should be interpreted as a transparent reference economy. Its role is to locate the inherited and observed market states relative to the theoretical thresholds and to quantify the general equilibrium channels that cannot be signed analytically.

5.2 Aggregate Demand, Production, and Ownership

Aggregate labor supply, the prices of the two digital services, inherited platform quality, and the pre-regulation wage are normalized to one:

$$\bar{L} = P_P = P_R = q_{P0} = w_0 = 1.$$

However, equal normalized prices do not imply that the digital services provided by the platform and the rival have the same quality-adjusted price, since $p_{j0} = P_j/q_{j0}$.

The inherited household digital expenditure share is constructed from BEA Table 2.4.5U (U.S. Bureau of Economic Analysis, 2026). I combine household expenditures on cable and live television, cellular telephone services, internet access, and video and audio streaming and rental. In 2023, these categories account for

$$S_0^H = 0.021807.$$

This moment disciplines the size of the broad household digital-service bundle rather than app-store revenue itself.

Because μ is a CES distribution parameter rather than an expenditure share, it is calculated from

$$\frac{\mu}{1 - \mu} = \left[\frac{S_0^H}{1 - S_0^H} P_{D0}^{\sigma-1} \right]^{1/\sigma}. \quad (114)$$

With $\sigma = 0.4$ and $P_{D0} = 1$, this gives $\mu = 7.4201 \times 10^{-5}$. The small numerical value reflects the CES parameterization and should not be interpreted as the digital expenditure share.

The benchmark sets $\sigma = 0.4$, allowing digital and non-digital consumption to be imperfect substitutes, and $\sigma_D = 4$, reflecting greater substitutability between two available app-distribution services than between the digital and non-digital bundles. Both elasticities are assigned and varied in the sensitivity analysis.

The digital intermediate-input share is set to $\iota = 0.30$. This deliberately high benchmark combines a conventional labor share of approximately 0.70 with the model's attribution of the remaining input share to the digital composite. It should be interpreted as an upper-bound benchmark for the contemporaneous downstream production channel, not as a direct estimate for app distribution.¹¹

Given $P_{D0} = w_0 = 1$, non-digital productivity follows from the unit-cost condition:

$$B = \left(\frac{w_0}{1 - \iota} \right)^{1-\iota} \left(\frac{P_{D0}}{\iota} \right)^\iota = 1.8420.$$

The foreign ownership share of platform profit is set to $\phi = 0.85$. Country-level portfolio holdings reported in the U.S. Treasury benchmark survey motivate an approximate domestic ownership share of 15 percent (U.S. Department of the Treasury, 2025). Because portfolio ownership is an imperfect proxy for the incidence of platform profits, ϕ is treated as an assigned distributional parameter.

¹¹The ecosystem estimates mentioned in the section 4 are used to motivate the presence and quantitative relevance of the downstream production linkage, rather than to calibrate ι directly. Sales facilitated through apps are gross transaction values and therefore do not measure either expenditure on digital intermediate inputs or the causal elasticity of non-digital output with respect to those inputs. I treat ι as an assigned sensitivity parameter rather than an estimated parameter. Future work could replace this upper-bound benchmark with a market-specific estimate constructed from input-output data.

5.3 Inherited Production Conditions

The effective inherited platform operating margin is set to $\nu_{P0} = 0.22$.¹² Using

$$\nu_{P0} = 1 - \frac{w_0}{P_P A_{P0}},$$

the normalization implies

$$A_{P0} = \frac{1}{1 - \nu_{P0}} = 1.2821.$$

I assign a modest inherited platform productivity advantage,

$$\frac{A_{R0}}{A_{P0}} = 0.95, \quad A_{R0} = 1.2179$$

The platform-margin and relative-productivity assumptions are constrained partly by operational viability: lower values can make an active challenger unable to cover variable production costs in some innovation states. Both are varied in the sensitivity analysis.

5.4 Effective Market Access, the Inherited Relative-Demand Position, and Entry

With inherited and observed effective market access weights measured empirically, I target a 2 percent rival share of digital expenditure at the observed market access state:

$$s_R^{\text{obs}} = 0.020 \quad \text{at} \quad \kappa_1^{\text{obs}} = 0.47.$$

Inherited rival quality q_{R0} is chosen so that expected period-1 challenger expenditure matches this target, conditional on entry and equilibrium innovation. This gives

$$q_{R0} = 0.4244.$$

In the formal model, q_{R0} is the rival's pre-innovation quality. In the quantitative application, it should be interpreted more broadly as the parameter through which the calibration matches the challenger's *inherited relative-demand position*. It absorbs demand-side competitive asymmetries not separately represented in the model, including possible differences in product characteristics, networks, data, reputation, complements, user familiarity, and accumulated product development. The calibration does not identify the historical sources of these asymmetries.¹³

The market share identity from Proposition 1 helps interpret this calibration. At the observed pair $(\kappa_1^{\text{obs}}, s_R^{\text{obs}}) = (0.47, 0.02)$, the implied relative-demand term is

$$\Omega^{\text{obs}} = \frac{0.02}{0.47(1 - 0.02)} \simeq 0.0434.$$

Holding the realized price-quality position fixed, the rival's market share at full market access would be

$$s_R^{(1)} = \frac{\Omega^{\text{obs}}}{1 + \Omega^{\text{obs}}} \simeq 0.0416.$$

¹²This is not a direct estimate of the App Store's accounting margin. It is the smallest rounded benchmark that preserves positive operating margins in every innovation state under the high-spillover specification and the assumed rival productivity gap.

¹³In particular, a low calibrated q_{R0} is not evidence that the challenger's inherited position is intrinsic, independent of earlier market access, or attributable to insufficient effort.

This accounting shows that removing the remaining modeled access disadvantage produces a rival share near 4 percent in the benchmark quantitative equilibrium.

The fixed entry cost is disciplined by inherited non-entry and observed post-implementation entry. Then we have

$$\bar{V}_R(\kappa_0) \leq F_R \leq \bar{V}_R(\kappa_1^{\text{obs}}).$$

The identified interval is

$$F_R \in [2.5538 \times 10^{-5}, 7.7713 \times 10^{-4}].$$

I select its midpoint $F_R = 4.0134 \times 10^{-4}$ for the benchmark. The resulting commercial-entry threshold is a model outcome rather than a direct calibration target.

5.5 Innovation Technology

The common innovation efficiency parameter η is calibrated to the firm-wide R&D intensity reported in Apple's fiscal-year 2023 Form 10-K (Apple Inc., 2023).¹⁴ Apple reports R&D expenditure of \$29.915 billion and net sales of \$383.285 billion:

$$\frac{\text{R\&D}}{\text{sales}} = \frac{29.915}{383.285} = 0.078049.$$

I choose η so that platform innovation expenditure relative to expected platform digital revenue matches the observed firm-wide R&D intensity in the inherited platform-only equilibrium. In the calibrated model, platform R&D investment is hump-shaped in η : greater innovation efficiency initially raises the return to R&D and increases optimal expenditure, but beyond the peak of the schedule, further efficiency gains reduce the expenditure required to attain high innovation-success probabilities.¹⁵

Consequently, the observed R&D-to-revenue target is matched by two values of the common innovation-efficiency parameter:

$$\eta \in \{128.9466, 207.6854\}.$$

The benchmark calibration uses the smaller solution, $\eta = 128.9466$, which lies on the locally increasing branch of the R&D-intensity schedule. However, this choice is a branch-selection convention rather than an empirical identification of the smaller root. I report the larger-root economy as an alternative calibration to assess the sensitivity of the quantitative conclusions to this multiplicity. An additional market-specific moment tied to innovation output, innovation-success probabilities, or the productivity of R&D would be required to distinguish the two branches empirically.

The benchmark sets

$$\lambda = 0.75, \quad \delta = 0.124, \quad \gamma_D = 2.5.$$

¹⁴Because Apple does not report R&D separately for app distribution, the firm-wide ratio is a proxy rather than a market-specific estimate.

¹⁵A one-direction benchmark illustrates the mechanism. Holding the expected operating-profit gain from a successful innovation, $\Delta\pi > 0$, fixed, an interior innovation choice satisfies $1 = \beta\eta a \exp(-\eta ax)\Delta\pi$. Optimal expenditure is therefore $x(\eta) = \frac{\log(\beta\eta a \Delta\pi)}{\eta a}$, with $\frac{\partial x(\eta)}{\partial \eta} = \frac{1 - \log(\beta\eta a \Delta\pi)}{a\eta^2}$. Innovation expenditure rises when $\log(\beta\eta a \Delta\pi) < 1$ and falls when $\log(\beta\eta a \Delta\pi) > 1$. Numerical evaluation of the full equilibrium confirms the same non-monotonic pattern: the platform R&D-to-revenue ratio crosses the target with a positive local slope at the smaller root and a negative local slope at the larger root.

Table 2: **Benchmark calibration**

Block	Parameter	Value	Discipline
Preferences	β	0.9216	Assigned two-year discount factor
	S_0^H	0.02181	2023 broad digital-expenditure target
	μ	7.4201×10^{-5}	Calculated from S_0^H
	σ	0.400	Assigned; sensitivity analysis
	σ_D	4.000	Assigned; sensitivity analysis
Production	ι	0.300	Upper-bound benchmark; sensitivity analysis
	B	1.8420	Wage and unit-cost normalization
	ν_{P0}	0.220	Operational-viability benchmark
	A_{P0}	1.2821	Calculated from ν_{P0}
	A_{R0}/A_{P0}	0.950	Assigned; sensitivity analysis
Market state	κ_0	0.0144	Measured inherited access state
	κ_1^{obs}	0.470	Measured observed access state
	s_R^{obs}	0.020	Observed challenger-share target
	q_{R0}	0.4244	Matches inherited relative-demand position
	ϕ	0.850	Ownership proxy; sensitivity analysis
	F_R	4.0134×10^{-4}	Midpoint of identified interval
Innovation	η	128.9468	Smaller root matching Apple firm-wide R&D intensity
	λ	0.750	Assigned; sensitivity analysis
	δ	0.124	Assigned; sensitivity analysis
	γ_D	2.500	Assigned; sensitivity analysis
	ζ_R	1.000	Benchmark inherited innovation efficiency

The comparative-advantage parameter gives the platform greater efficiency in incremental innovation and the challenger greater efficiency in disruptive innovation. An incremental success improves quality and productivity by $\exp(0.124) - 1 = 13.2$ percent. A disruptive success improves them by $\exp(2.5 \times 0.124) - 1 = 36.3$ percent. These values capture the distinction between incremental and more radical innovation emphasized in the innovation literature (Acemoglu and Cao, 2015; Akcigit and Kerr, 2018; Caggese, 2019; Klenow and Li, 2021). Their exact magnitudes are assigned and varied in the sensitivity analysis.

The empirical benchmark normalizes challenger-specific innovation efficiency to $\zeta_R = 1$, which imposes no additional firm-specific innovation-efficiency advantage. A separate exercise sets $\zeta_R = 2.5$ to examine how greater inherited innovation capacity changes the innovation threshold. In the sensitivity analysis, rival success probabilities are instead written as $\theta_{Rk} = 1 - \exp(-\eta \zeta_R a_{Rk} x_{Rk})$, and $\zeta_R > 1$. This extension allows the quantitative exercise to distinguish between a lack of commercial scale and a lack of rival R&D productivity.

5.6 Calibration Summary and Numerical Discipline

Table 2 summarizes the benchmark parameters. The model matches the household digital-expenditure share, platform R&D intensity, and observed challenger share to numerical precision. The selected entry cost satisfies inherited non-entry and observed post-implementation entry. Across the simulated access grid, reported general-equilibrium residuals are below 4.5×10^{-16} , best-response errors are below 5.7×10^{-9} , and every active equilibrium state satisfies operational viability and transition feasibility.

5.7 Counterfactual Design and Welfare

The benchmark calibration is the reference economy for the quantitative analysis. It matches the inherited no-entry and the observed post-DMA rival entry and market share. The main counterfactual asks what improved effective market access can accomplish starting from that inherited competitive state. I hold the inherited market environment $\mathcal{H}_0$, the fixed entry cost, and all innovation and general equilibrium parameters at their benchmark values and vary only κ_1 over $[\kappa_0, 1]$. Rival entry, both firms' innovation choices, and all general-equilibrium outcomes are re-solved at every value of κ_1 . The benchmark market-access path contains the three reference states in Table 3.

Table 3: Reference States on the Benchmark Market-Access Path

Reference state	Effective market access	Role in the analysis
No-improvement state	$\kappa_1 = \kappa_0 = 0.0144$	Preserves inherited market access and provides the benchmark for entry, innovation, and welfare comparisons.
Observed post-DMA state	$\kappa_1 = \kappa_1^{\text{obs}} = 0.47$	Locates the observed market relative to the commercial-entry and rival-innovation thresholds.
Full-access benchmark	$\kappa_1 = 1$	Removes the contemporaneous market-access disadvantage represented by κ_1 , conditional on the inherited competitive state and the adjustment mechanisms in the two-period model.

Tracing the entire path identifies the commercial-entry threshold and any rival-innovation threshold and shows how market shares, innovation, profits, and welfare evolve as current market access improves. The full market access benchmark is a conditional transition outcome. It is not a prediction of the long-run market structure that would emerge after repeated network formation, data accumulation, complementary entry, reinvestment, and innovation.

Complementary rival paths. The benchmark access path holds inherited competition conditions fixed and varies only the effective market access κ_1 . As such, it cannot show how the results would differ if rivals arrived at the transition with a stronger inherited position or a more productive innovation technology. I use the two complementary structural exercises to study those questions.

The first is the *less-disadvantaged rival path*, where the rival's inherited quality and productivity gaps are progressively narrowed, while leaving all other benchmark parameters unchanged.

Define the benchmark inherited quality and productivity gaps as

$$G_q^B = q_{P0} - q_{R0}^B, \quad G_A^B = A_{P0} - A_{R0}^B.$$

For $\rho \in [0, 1]$, the path closes the common fraction ρ of each gap:

$$q_{R0}(\rho) = q_{R0}^B + \rho G_q^B,$$

$$A_{R0}(\rho) = A_{R0}^B + \rho G_A^B.$$

Equivalently,

$$\frac{q_{R0}(\rho) - q_{R0}^B}{q_{P0} - q_{R0}^B} = \frac{A_{R0}(\rho) - A_{R0}^B}{A_{P0} - A_{R0}^B} = \rho.$$

Thus, ρ is the fraction of each benchmark gap that is closed.¹⁶

The second is the *higher rival innovation-capacity path*. It raises the rival-specific innovation-efficiency multiplier from $\zeta_R = 1$ to $\zeta_R = 2.5$. Because the purpose is to compare markets with the same observed commercial foothold but different abilities to convert that foothold into innovation, the inherited demand shifter and fixed entry cost are recalibrated to preserve the observed 2 percent share, inherited non-entry, and observed entry.

Table 4 distinguishes what changes, what is recalibrated, and what remains fixed in the two exercises.

Table 4: Complementary Rival Paths Relative to the Benchmark Calibration

Design feature	Less-Disadvantaged Rival Path	Higher Rival Innovation-Capacity Path
Direct change	q_{R0} and A_{R0} move from their benchmark values toward the platform's values as ρ rises from zero to one.	Rival innovation efficiency rises from $\zeta_R = 1$ to $\zeta_R = 2.5$.
Recalibrated objects	None.	q_{R0} is recalibrated to preserve the observed 2 percent rival share. F_R is then reselected from the interval consistent with inherited non-entry and observed entry.
Held at benchmark values	$\zeta_R = 1$, F_R , all platform characteristics, all other innovation parameters, preferences, production parameters, posted prices, κ_0 , and κ_1^{obs} .	All parameters other than ζ_R , q_{R0} , and F_R . In particular, A_{R0} , platform characteristics, the common innovation parameter η , preferences, production parameters, posted prices, and measured access states are unchanged.
Observed 2 percent share	Imposed only at $\rho = 0$ and allowed to respond endogenously for $\rho > 0$.	Preserved through the recalibration of q_{R0} .
Equilibrium calculation	The full market-access path is re-solved for every pair (ρ, κ_1) .	After recalibration, the full market-access path over $\kappa_1 \in [\kappa_0, 1]$ is re-solved.
Question answered	How do entry and innovation thresholds and market structure change as inherited quality and productivity gaps narrow?	Among markets with the same observed foothold and entry pattern, how does greater rival innovation capacity change dynamic outcomes?

Notes: Superscript B denotes a benchmark-calibration value. The term “less disadvantaged” describes a narrower inherited quality and productivity gap within the model. It does not identify the historical cause of that gap.

For the cross-path comparisons in Subsection 6.5, I use the less-disadvantaged economy at $\rho = \rho^{I,\text{obs}} = 0.2262$, the point at which observed market access lies on the rival-innovation boundary. Selecting this value turns the two-dimensional (ρ, κ_1) exercise into one market-access

¹⁶The exercise does not interpret ρ as a common percentage increase in the levels of quality and productivity.

path that can be compared directly with the benchmark and higher-capacity paths. The full results of less-disadvantaged-rival economies are reported separately in the quantitative results.

Welfare measurement and decomposition. Welfare is measured relative to the no-improvement equilibrium within each path. Let $W_j(\kappa_1)$ denote expected two-period household utility under path $j \in \{B, L, H\}$, where B denotes the benchmark, L the selected less-disadvantaged-rival path, and H the higher-innovation-capacity path. The consumption-equivalent welfare change is

$$\Delta CE_j(\kappa_1) = \exp \left\{ \frac{W_j(\kappa_1) - W_j(\kappa_0)}{1 + \beta} \right\} - 1. \quad (115)$$

Thus, welfare comparisons along each path isolate the value of improving market access given that path's inherited state and calibration. In particular, the less-disadvantaged path does not treat the change in ρ itself as a costless policy intervention.

To distinguish commercialization and associated general-equilibrium effects from endogenous innovation, I also solve a fixed-innovation counterfactual within each path. Entry and all general-equilibrium allocations respond to κ_1 , but firms' innovation expenditures remain fixed at the no-improvement level for that path. Let $W_j^C(\kappa_1)$ denote utility in that counterfactual. Then

$$\begin{aligned} W_j(\kappa_1) - W_j(\kappa_0) = & \underbrace{W_j^C(\kappa_1) - W_j(\kappa_0)}_{\text{commercialization and associated general-equilibrium effect}} \\ & + \underbrace{W_j(\kappa_1) - W_j^C(\kappa_1)}_{\text{endogenous innovation effect}}. \end{aligned} \quad (116)$$

The decomposition is additive in utility. The corresponding consumption-equivalent percentages need not add exactly because the consumption-equivalent transformation is nonlinear.

6 Quantitative Results

The quantitative results follow the design in Section 5.7. The benchmark path is the main quantitative exercise. It varies only the effective market access κ_1 , while holding the inherited competitive state, the entry cost, and all preference, production, and innovation parameters at their benchmark values. I first use that path to study commercial entry, commercialization, and rival innovation. I then use the share equation to measure the accounting distance from the observed market to a 50–50 market share reference. The next two subsections examine how the entry and innovation thresholds change when the rival starts from a less disadvantaged inherited position or the rival has greater innovation capacity. The final subsection compares innovation and welfare across all three paths.

6.1 Benchmark Market-Access Path: Entry, Commercialization, and Innovation

Along the benchmark economy path, only the effective market access κ_1 varies. Rival entry, both firms' innovation choices, and the full state-contingent general equilibrium are re-solved at every value of κ_1 .

Commercial entry. The rival's net entry value crosses zero at $\kappa^E = 0.2342$. Relative to inherited effective market access, the minimum improvement required for entry is therefore

$$\Delta\kappa^E = 0.2342 - 0.0144 = 0.2198.$$

The observed post-regulation implementation state, $\kappa_1^{\text{obs}} = 0.47$, lies well above this threshold. The benchmark economy has therefore moved from Regime I, in which the rival does not enter, into Regime II, in which entry is commercially viable but rival innovation remains inactive.

Figure 3 plots the actual, unnormalized net entry value. It is increasing over the reported grid, crosses zero once, and remains positive thereafter. The benchmark numerical economy therefore satisfies the single-threshold entry-support condition in Proposition 2.

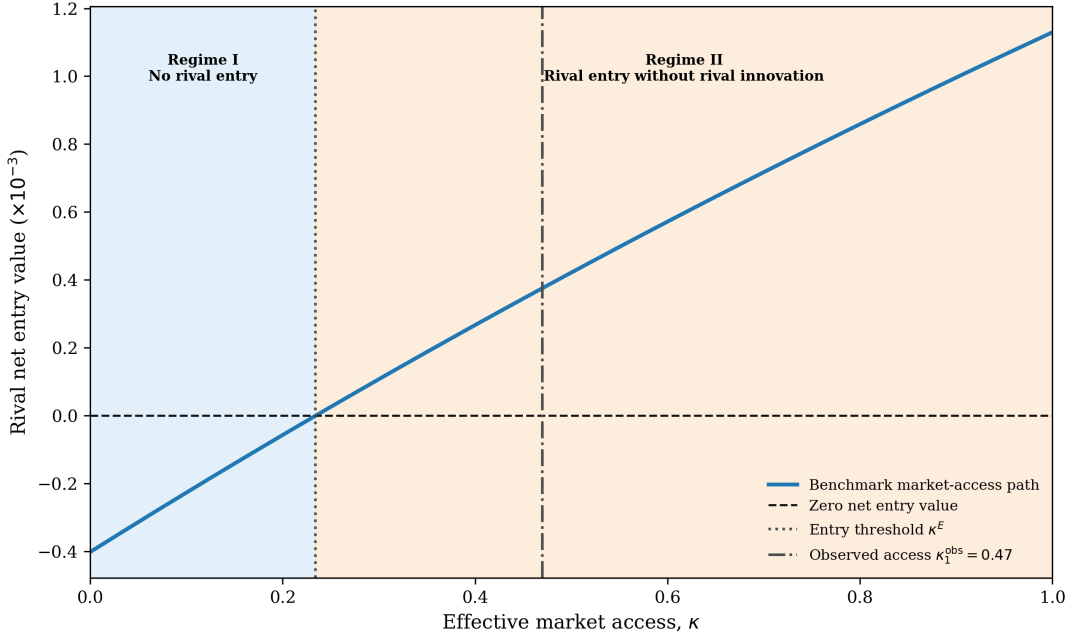


Figure 3: **Benchmark net entry value and entry regimes.** The horizontal dashed line marks zero net entry value. The dotted vertical line marks the commercial-entry threshold, and the dash-dotted line marks observed effective market access. The blue and light-orange backgrounds identify Regimes I and II. Values on the vertical axis are expressed in 10^{-3} units of the numeraire.

Commercialization after entry. Entry changes discontinuously at κ^E , whereas the rival's expenditure share changes continuously with market access conditional on entry. Below the threshold, greater access raises the sales the rival could obtain if it entered, but the value of that commercial opportunity remains insufficient to cover the fixed entry cost. At the threshold, the realized rival market share jumps from zero to a positive level. Above it, further access improvements expand the commercial scale of an active rival.

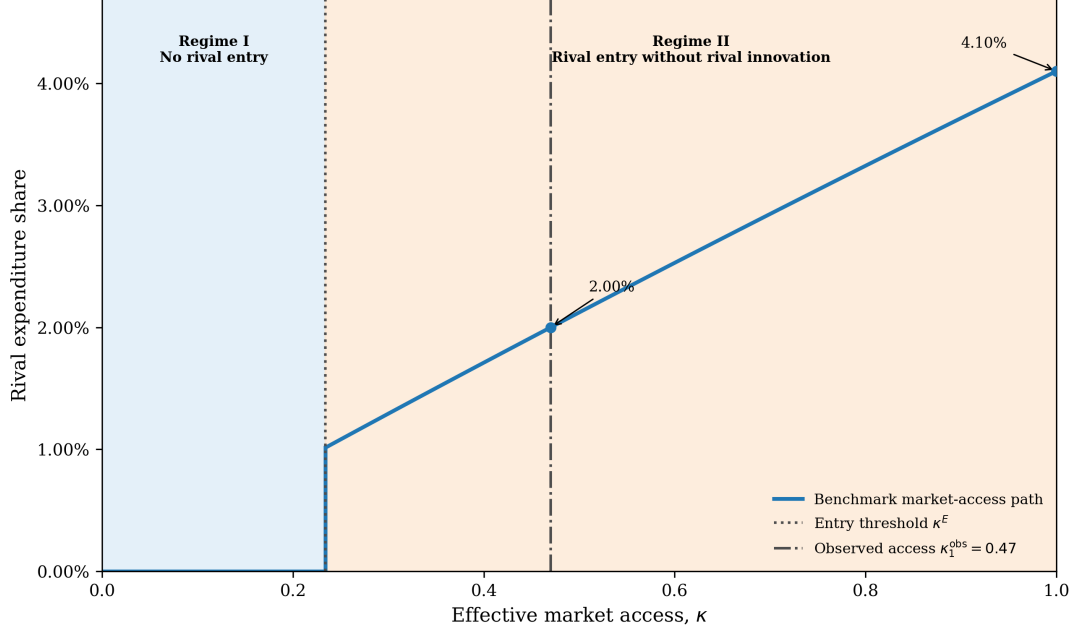


Figure 4: **Rival commercialization along the benchmark market-access path.** Below κ^E , the rival remains inactive, and its realized expenditure share is zero. At the threshold, entry becomes viable, and the realized market share jumps to a positive level. Conditional on entry, rival share rises continuously with effective market access, reaching 2 percent at observed access and 4.10 percent at full access. The benchmark remains in Regime II after entry.

Table 5 summarizes the market structure and rival innovation outcomes at the three reference states. Platform innovation and aggregate outcomes are deferred to Subsection 6.5, where they are compared across all three paths.

Table 5: Benchmark Market Structure and Rival Innovation at the Reference States

Outcome	Inherited: $\kappa_0 = 0.0144$	Observed: $\kappa_1 = 0.47$	Full access: $\kappa_1 = 1$
Rival enters	No	Yes	Yes
Rival regime	I	II	II
Rival expenditure share (%)	0.000	2.000	4.101
Digital price index	0.8107	0.8018	0.7927
Rival incremental investment	0.00000	0.00000	0.00000
Rival disruptive investment	0.00000	0.00000	0.00000
Incremental marginal benefit/cost	—	0.043	0.086
Disruptive marginal benefit/cost	—	0.485	0.949

Notes: Rival marginal-benefit-to-cost ratios are evaluated at zero investment, conditional on entry. A ratio above one makes initiating the corresponding innovation investment privately profitable. Dashes indicate that the rival is inactive.

The 4.10 percent market share at full market access is a conditional transition result. The counterfactual removes the contemporaneous market-access disadvantage represented by κ_1 but retains the inherited competitive position used to match the observed market and permits only the one-round innovation response contained in the two-period model. It is not a prediction of the long-run share that would emerge after repeated user acquisition, network formation, data accumulation, and innovation.

The digital price index nevertheless falls by 1.10 percent at observed access and by 2.22 percent at full access relative to the no-improvement equilibrium. Clearly, even a relatively small rival can lower the quality-adjusted price of digital services even when inherited competitive conditions prevent a large market-share reallocation over the model's horizon.

The reported magnitudes are full-equilibrium outcomes and also incorporate the platform's endogenous innovation response. The broader consequences of the lower digital price index for household purchasing power, digital-input costs, wages, non-digital production, and welfare are examined across all three counterfactual paths in Subsection ??.

Rival innovation: entry does not yet produce dynamic competition. The benchmark's central rival innovation result is a corner solution. The rival enters and acquires positive sales, but it invests in neither incremental nor disruptive innovation at any $\kappa_1 \in [\kappa_0, 1]$. This outcome is not a numerical failure. It means that the commercial scale generated by the benchmark path is insufficient to make rival innovation privately profitable.

To measure the distance from the innovation boundary, define the rival's marginal benefit-to-cost ratio at zero investment in direction $k \in \{I, D\}$:

$$\mathcal{R}_{Rk}(\kappa) \equiv \beta \eta a_{Rk} \mathbb{E}_{\xi_{-Rk}} [\Delta \pi_{Rk}(\xi_{-Rk}; \kappa)].$$

Because the marginal resource cost of innovation is normalized to one, the rival begins investing when $\mathcal{R}_{Rk}(\kappa) > 1$ and remains at the zero boundary when $\mathcal{R}_{Rk}(\kappa) \leq 1$.

At observed effective market access,

$$\mathcal{R}_{RI}(0.47) = 0.043, \quad \mathcal{R}_{RD}(0.47) = 0.485.$$

Thus, the marginal benefit of disruptive innovation covers about 48.5 percent of its marginal cost, while the return to incremental innovation is much lower. At full market access,

$$\mathcal{R}_{RI}(1) = 0.086, \quad \mathcal{R}_{RD}(1) = 0.949.$$

The disruptive return comes close to, but does not cross, the investment boundary. Incremental innovation remains much farther away, consistent with the rival's comparative advantage in disruptive innovation and the larger technological step associated with disruption.

Figure 5 traces both marginal-benefit-to-cost ratios conditional on entry for each innovation direction. Both ratios rise with access, but neither reaches one. The benchmark therefore moves from Regime I to Regime II at κ^E and remains in Regime II through the full-access benchmark.

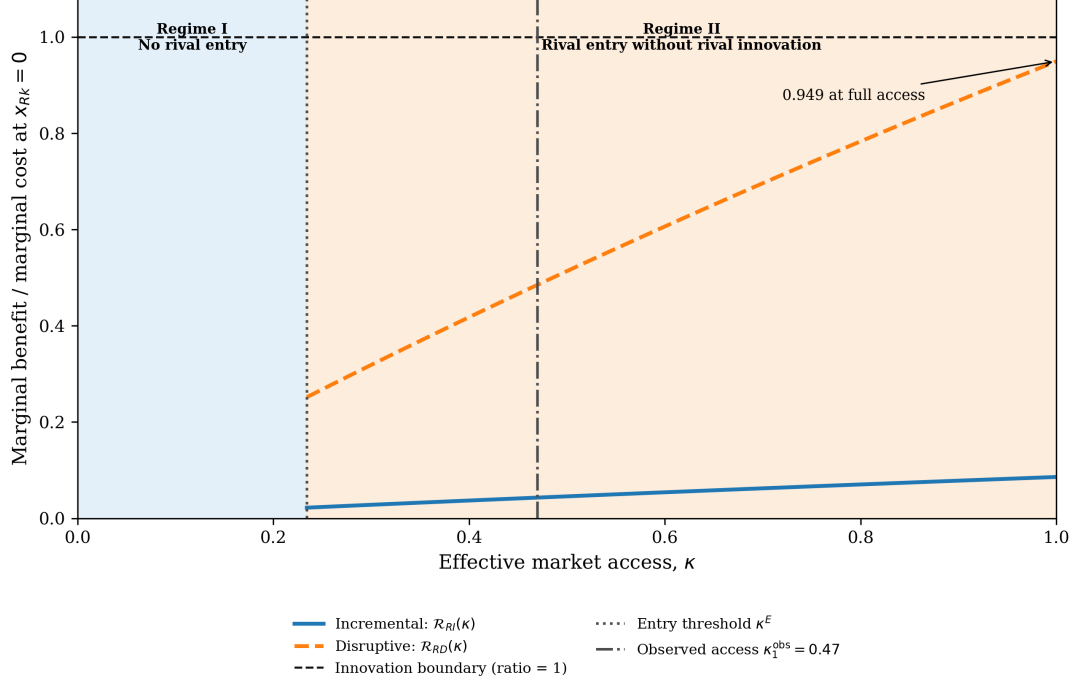


Figure 5: **Rival innovation incentives along the benchmark market-access path.** The two curves report the marginal benefit of initiating incremental and disruptive rival innovation, respectively, divided by their common unit marginal cost. Each is evaluated at zero investment in its own direction. The horizontal line at one is the innovation boundary. The disruptive return reaches 0.949 at full access but does not cross the boundary, so Regime III is not reached.

This result is conditional on the inherited competitive state. It does not imply that the rival's limited ability to earn innovation returns is intrinsic or independent of earlier market closure. The inherited demand, productivity, network, data, reputation, and product-development position may itself partly reflect the preceding access regime.

6.2 Accounting Distance from the Observed Market to the 50–50 Reference

The benchmark results raise a natural question. The rival receives 2 percent of digital expenditure at observed effective market access and only about 4 percent when the remaining contemporaneous access gap is removed. How far is the observed market from the 50–50 market share reference, and how much of that accounting distance is represented by current market access rather than the inherited relative-demand position? The share-odds relationship provides an exact accounting answer:

$$\frac{s_R}{1 - s_R} = \kappa \Omega.$$

At the observed pair $(\kappa_1^{\text{obs}}, s_R^{\text{obs}}) = (0.47, 0.02)$, the share-equivalent relative-demand term is

$$\Omega^{\text{obs}} = \frac{s_R^{\text{obs}}}{\kappa_1^{\text{obs}} (1 - s_R^{\text{obs}})} = \frac{0.02}{0.47(0.98)} = 0.04342.$$

Holding this term fixed and setting $\kappa = 1$ (full market access) gives

$$s_R(1) = \frac{0.04342}{1 + 0.04342} = 0.04161,$$

or 4.16 percent. This direct accounting result is almost identical to the 4.10 percent market share generated by the benchmark equilibrium.

To give the observed rival share an intuitive scale, consider a reference case in which the platform and rival have the same quality-adjusted price, so that $\Omega = 1$. Under this condition, the rival's expenditure share becomes equation $s_R(\kappa) = \frac{\kappa}{1+\kappa}$, and the rival's share reaches one-half at full market access, $\kappa = 1$.

I use this 50–50 endpoint as an accounting reference rather than as an alternative empirical calibration, policy target, or prediction of the long-run market structure.¹⁷

The observed rival-platform market share ratio (rival-share odds) is $\frac{0.02}{0.98} = 0.02041$. However, reaching 50–50 reference requires the rival-share odds to increase by a factor of $\frac{1}{0.02041} = 49$. Removing the remaining current market-access gap multiplies the odds by $\frac{1}{\kappa_1^{\text{obs}}} = \frac{1}{0.47} = 2.13$, while closing the inherited relative-demand gap multiplies the odds by $\frac{1}{\Omega^{\text{obs}}} = 23.03$.

Equivalently, the total distance is additive in log odds:

$$\underbrace{-\log\left(\frac{s_R^{\text{obs}}}{1-s_R^{\text{obs}}}\right)}_{\text{total distance to the 50–50 reference}} = \underbrace{-\log \kappa_1^{\text{obs}}}_{\text{current market-access component}} + \underbrace{-\log \Omega^{\text{obs}}}_{\text{inherited relative-demand component}}$$

$$3.8918 = 0.7550 + 3.1368.$$
(117)

Thus, the remaining current market-access gap represents 19.4 percent of the total accounting distance in log-odds terms, while the inherited relative-demand term represents 80.6 percent. Figure 6 visualizes this exact decomposition.

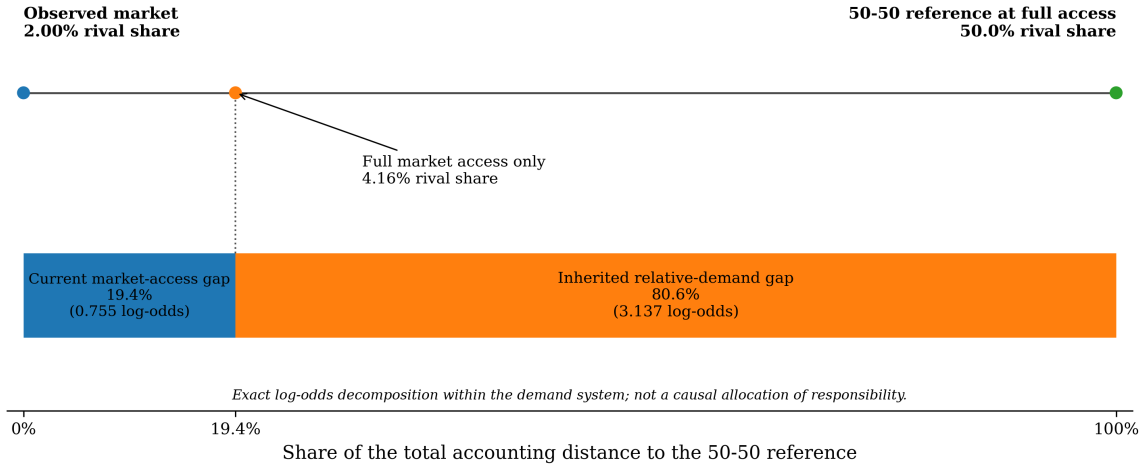


Figure 6: Accounting decomposition of the observed rival-share gap. The total log-odds distance from the observed 2 percent rival share to the 50–50 reference is normalized to 100 percent. The remaining current market-access gap represents 19.4 percent and raises the implied share to 4.16 percent when removed. The remaining 80.6 percent is represented by the inherited relative-demand term. This is an accounting identity within the demand system, not a causal allocation of responsibility.

The decomposition does not imply that current access is already adequate or that the remaining gap is attributable to insufficient rival effort. The relative-demand component may include contemporaneous product and preference differences, but it may also embody accumulated differences in networks, scale, data, reputation, complementary services, user familiarity,

¹⁷More generally, equal expenditure shares require $\kappa\Omega = 1$. Equalizing quality-adjusted prices while retaining observed access would imply a 31.97 percent rival share.

and product-development opportunities. Some of those differences may themselves reflect the earlier market-access regime. Nor is a 50–50 split necessary for effective or dynamic competition. The structural exercises below ask the more relevant question: how much must inherited competitive conditions change before rival innovation becomes viable?

6.3 Less-Disadvantaged Rival Path

The benchmark rival remains in Regime II even at full access, but its marginal-benefit-to-cost ratio for disruptive innovation reaches 0.949. This raises a natural question: how much stronger would the rival’s inherited quality and productivity position need to be for Regime III to become attainable?

As defined in Section 5.7,

$$\begin{aligned} q_{R0}(\rho) &= (1 - \rho)q_{R0}^B + \rho q_{P0} = q_{R0}^B + \rho (q_{P0} - q_{R0}^B), \\ A_{R0}(\rho) &= (1 - \rho)A_{R0}^B + \rho A_{P0} = A_{R0}^B + \rho (A_{P0} - A_{R0}^B). \end{aligned}$$

By moving ρ from zero to one, we are closing inherited quality and productivity gaps, which means q_{R0} and A_{R0} change along this path. No other object is recalibrated. The observed 2 percent rival share is imposed only at $\rho = 0$ and is allowed to respond endogenously thereafter.

A higher q_{R0} strengthens the rival’s demand position at any given level of effective market access, while a higher A_{R0} improves its operating profit. Both changes raise the value of entry and the expected profit gain from successful innovation. The commercial entry threshold $\kappa^E(\rho)$ and rival-innovation threshold $\kappa^I(\rho)$ both decline as ρ increases.

When does rival innovation first become attainable? At $\rho = 0$, no rival-innovation threshold exists within the feasible market-access range $[\kappa_0, 1]$. The market moves from Regime I to Regime II at $\kappa^E = 0.2342$, but it never reaches Regime III.

However, closing only 1.51 percent of each benchmark quality and productivity gap is sufficient to bring the innovation threshold to the full-access boundary. Specifically,

$$\rho^{I,FA} = 0.0151 \quad \implies \quad \kappa^I(\rho^{I,FA}) = 1.$$

The more policy-relevant comparison is the observed effective market access. The rival-innovation threshold can be reduced to $\kappa^{\text{obs}} = 0.47$ when

$$\rho^{I,obs} = 0.2262 \quad \implies \quad \kappa^I(\rho^{I,obs}) = 0.47.$$

At this point, the commercial-entry threshold has already fallen to

$$\kappa^E(\rho^{I,obs}) = 0.0961. \tag{118}$$

The observed market is well inside the commercial-entry region and lies on the Regime-II/Regime-III boundary. In this calibration, disruptive innovation is the first rival innovation direction to become viable: its marginal benefit equals its marginal cost at the boundary, while incremental innovation remains below its profitability threshold. For any value of ρ slightly above 0.2262, observed effective market access supports both rival entry and rival innovation.

Figure 7 shows the full regime map.

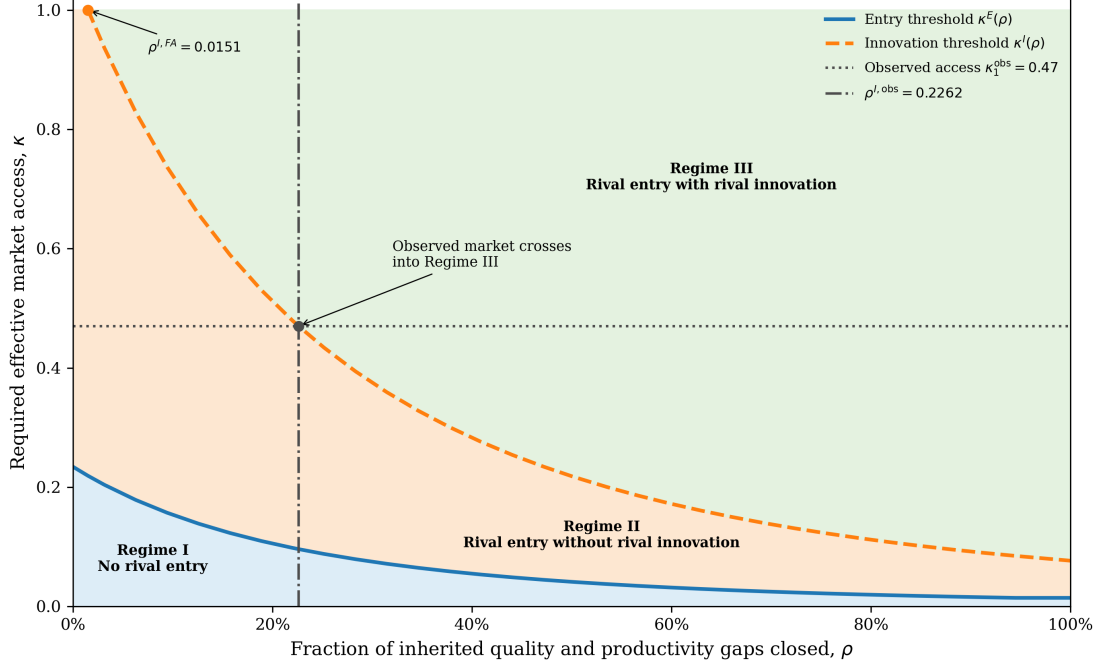


Figure 7: **Entry and innovation regimes along the less-disadvantaged rival path.** Regime I is the light-blue region below the entry threshold; Regime II is the light-orange region between the entry and innovation thresholds; and Regime III is the light-green region above the innovation threshold. The innovation threshold first enters the feasible range at $\rho^{I,FA} = 0.0151$ and reaches observed access at $\rho^{I,obs} = 0.2262$.

Table 6 reports selected points on the path after the principal threshold movements have been established.

Table 6: Thresholds and Regimes Along the Less-Disadvantaged Rival Path

Position on the path	ρ	q_{R0}	A_{R0}/A_{P0}	κ^E	κ^I	Regime at κ_1^{obs}
Benchmark rival	0.0000	0.4244	0.9500	0.2342	–	Regime II
Innovation threshold reaches full access	0.0151	0.4331	0.9508	0.2190	1.0000	Regime II
Innovation threshold reaches observed access	0.2262	0.5546	0.9613	0.0961	0.4700	Regime II/III boundary
Half of inherited gaps narrowed	0.5000	0.7122	0.9750	0.0413	0.2183	Regime III
Expected full-access rival share reaches 50%	0.9440	0.9678	0.9972	0.01443	0.0849	Regime III

Notes: A dash indicates that rival innovation remains at the zero boundary throughout the feasible market-access interval $[\kappa_0, 1]$. At an innovation threshold, the net marginal return equals the marginal cost.

Market structure around the innovation threshold. At $\rho^{I,obs} = 0.2262$, the rival's expenditure share at observed market access is only

$$s_R(0.47; \rho^{I,obs}) = 4.29\%.$$

The rival can reach the boundary of dynamic viability while remaining much smaller than the platform. At the same inherited position, its market share at full access is 12.90 percent, and its probability of disruptive success reaches 44.44 percent. When one-half of each benchmark gap is closed, observed market access lies clearly in Regime III: rival share is 12.91 percent, and the probability of disruptive success is 45.03 percent.

Figure 8 shows how the rival market share at observed and full access changes along the full ρ path.

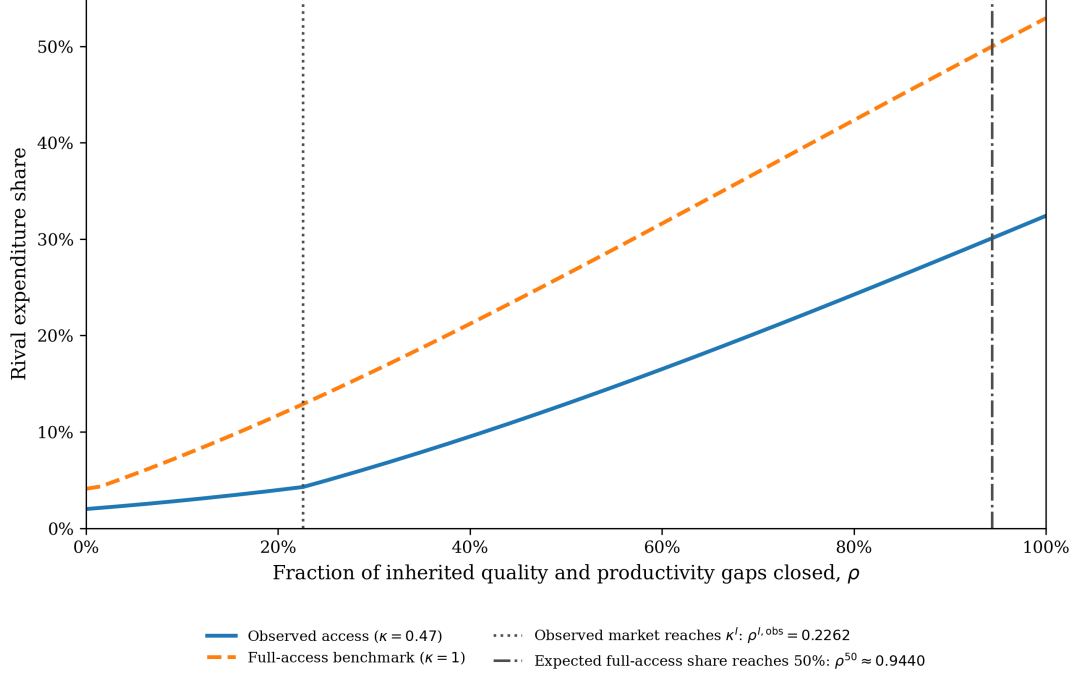


Figure 8: **Rival market shares along the less-disadvantaged rival path.** The curves report expected rival market shares at observed access and at the full-access benchmark. The first vertical marker identifies the point at which the observed market reaches the innovation threshold. The second marks the point at which the expected full-access rival share reaches 50 percent. Rival innovation becomes viable before the market approaches equal shares.

Equal shares and the inherited-entry boundary. A 50 percent rival share at full market access is reached only at

$$\rho^{50} = 0.9440, \quad s_R(1; \rho^{50}) = 0.50.$$

Thus, 94.4 percent of each benchmark quality and productivity gap must be closed along this particular path before the full-access equilibrium produces equal expected shares. This occurs before exact pre-innovation parity, $\rho = 1$, because the rival's comparative advantage in disruptive innovation strengthens its expected post-innovation position.

The nearby quantity $\rho^{E,0}$ has a different interpretation. Define

$$\rho^{E,0} \equiv \inf \{ \rho \in [0, 1] : \kappa^E(\rho) \leq \kappa_0 \}.$$

Numerically,

$$\rho^{E,0} = 0.9450, \quad \kappa^E(\rho^{E,0}) = \kappa_0,$$

This means that closing 94.5 percent of each benchmark inherited gap would make entry just viable already at the inherited access state. For any larger ρ , the maintained platform-only inherited equilibrium would fail. The 50–50 point is therefore a diagnostic boundary rather than an alternative empirically admissible calibration.

This path does not identify a policy instrument that changes ρ , the time required for inherited networks and capabilities to adjust, or the resources required for that adjustment beyond the entry and innovation costs already represented. It shows how inherited quality and productivity alter the access thresholds for entry and innovation.

6.4 Higher Rival Innovation-Capacity Path

The higher rival innovation-capacity path changes a different object. Rival innovation efficiency rises from $\zeta_R = 1$ to $\zeta_R = 2.5$. The inherited demand shifter q_{R0} and fixed entry cost F_R are recalibrated so that the alternative economy continues to match the observed 2 percent rival share, inherited non-entry, and observed post-implementation entry. The rival's inherited productivity and all other preference, production, access, and innovation parameters remain at their baseline values.

This exercise compares two calibrated economies with the same observed commercial foothold and entry pattern but different abilities to convert that foothold into successful innovation.

The recalibration gives

$$q_{R0} = 0.4084, \quad F_R = 3.6748 \times 10^{-4}.$$

The resulting thresholds are

$$\kappa_{\zeta}^E = 0.2398, \quad \kappa_{\zeta}^I = 0.4284.$$

This path moves from Regime I to Regime II at κ_{ζ}^E and from Regime II to Regime III immediately above κ_{ζ}^I . Because $\kappa_1^{\text{obs}} = 0.47$, the observed market lies in Regime III.

At observed access, $\kappa_1^{\text{obs}} = 0.47$, disruptive investment is 3.53×10^{-4} and the success probability is 8.19 percent. At full market access, disruptive investment rises to 0.00313, the success probability reaches 53.06 percent, and the rival share reaches 6.30 percent.¹⁸ Table 7 reports these outcomes in detail.

Table 7: Higher Rival Innovation-Capacity Path at the Reference States

Outcome	Inherited: $\kappa_0 = 0.0144$	Observed: $\kappa_1 = 0.47$	Full access: $\kappa_1 = 1$
Rival enters	No	Yes	Yes
Rival regime	I	III	III
Rival expenditure share (%)	0.000	2.000	6.303
Rival incremental investment	0.00000	0.00000	0.00000
Rival disruptive investment	0.00000	0.00035	0.00313
Disruptive success probability (%)	0.00	8.19	53.06

Figure 9 compares the benchmark and higher-capacity commercialization paths. They match a 2 percent rival share at observed access by construction. Beyond the innovation threshold, the higher-capacity rival improves its expected quality and productivity, so its share grows more rapidly.

¹⁸The commercial-entry threshold is close to the benchmark value, while the dynamic outcome changes substantially. This comparison should not be read as a pure comparative static of ζ_R , because q_{R0} and F_R are recalibrated. Its point, instead, is that the same observed 2 percent share and entry pattern can be consistent with very different innovation prospects.

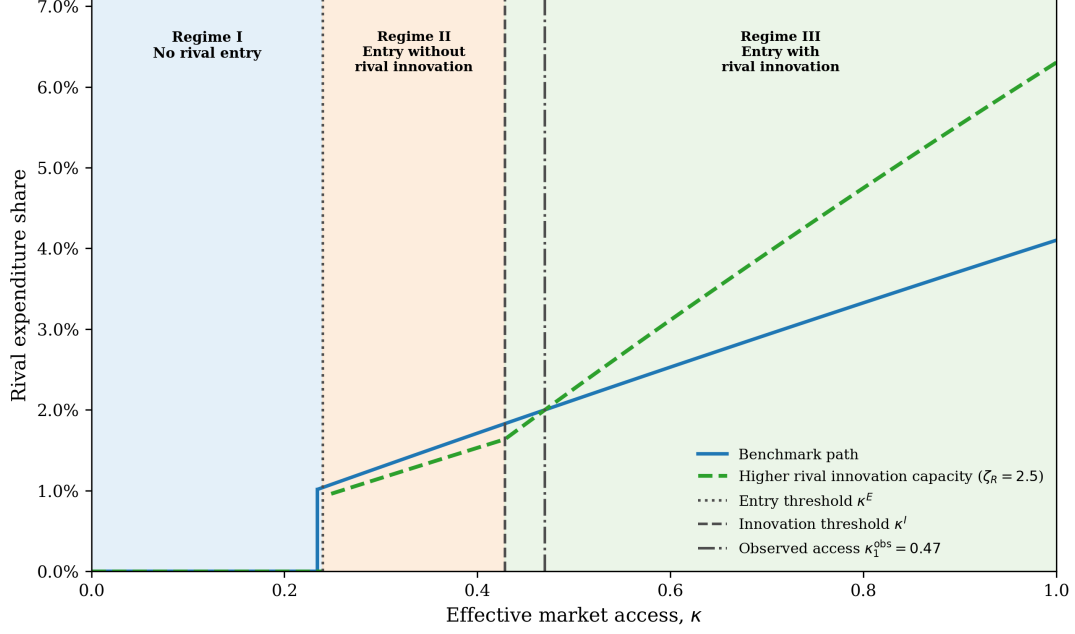


Figure 9: **Commercialization under higher rival innovation capacity.** The background regimes are defined by the higher-capacity thresholds. Both paths match a 2 percent rival share at observed access. The higher-capacity path enters Regime III above $\kappa_{\zeta}^I = 0.4284$ and reaches a 6.30 percent market share at the full-access benchmark, compared with 4.10 percent in the benchmark economy.

Figure 10 reports the actual net marginal return to rival disruptive innovation at the zero boundary. The benchmark return remains negative through full access, whereas the higher-capacity return crosses zero at κ_{ζ}^I .

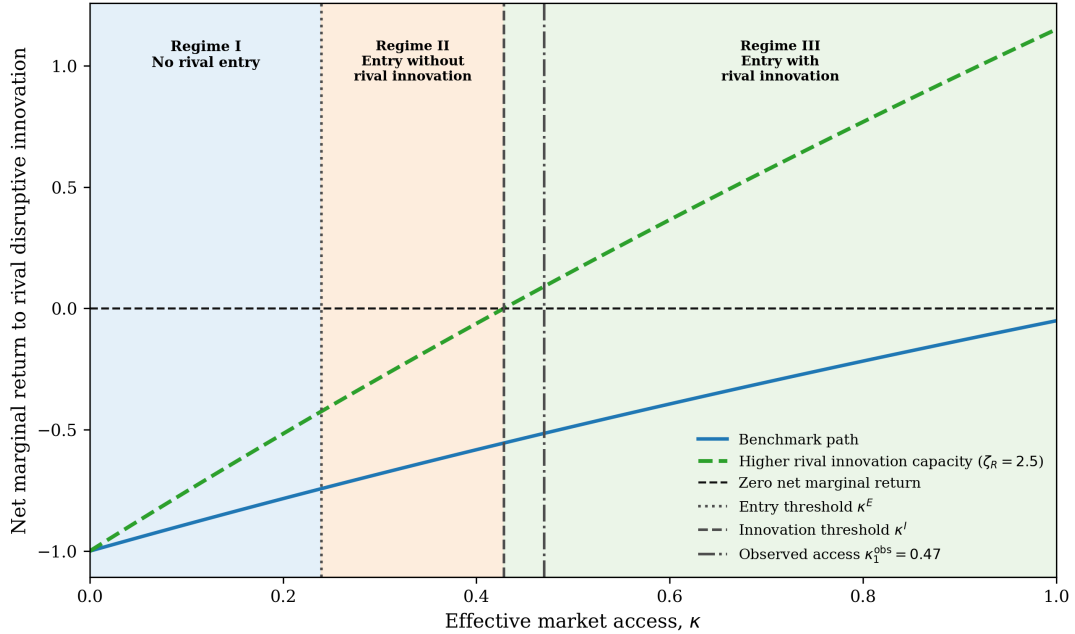


Figure 10: **Net marginal return to rival disruptive innovation.** A value below zero implies that zero disruptive investment is optimal at the margin. The benchmark path remains below zero throughout the feasible access range. The higher-capacity path crosses zero at its innovation threshold and enters Regime III.

The two complementary rival exercises activate innovation through different channels. The less-disadvantaged path improves the quality and productivity position from which the rival enters while holding all other primitives fixed. The higher-capacity path increases the productivity of R&D while recalibrating q_{R0} and F_R to preserve the same observed foothold. Both confirm that effective market access and inherited rival capacity jointly determine whether commercialization becomes dynamic competition.

6.5 Innovation and Welfare Across the Three Paths

This subsection compares three access paths on a common horizontal axis: (i) the benchmark path; (ii) a selected less-disadvantaged-rival path with $\rho = \rho^{I, \text{obs}} = 0.2262$; and (iii) the higher-rival-innovation-capacity path with $\zeta_R = 2.5$. The selected value of ρ is the point at which observed access lies exactly on the Regime-II/Regime-III boundary. It provides a transparent cross-path comparison, but it is only one point from the full family of less-disadvantaged-rival economies studied above.

Rival innovation across the three paths. The three paths generate sharply different rival-innovation outcomes. The benchmark rival remains in Regime II throughout the feasible access range and never innovates. The selected less-disadvantaged rival is exactly at the innovation boundary at observed access and begins disruptive innovation for slightly higher κ ; its disruptive-success probability reaches 44.44 percent at full access. The higher-capacity rival is already in Regime III at observed access, with an 8.19 percent success probability, and reaches 53.06 percent at full access.

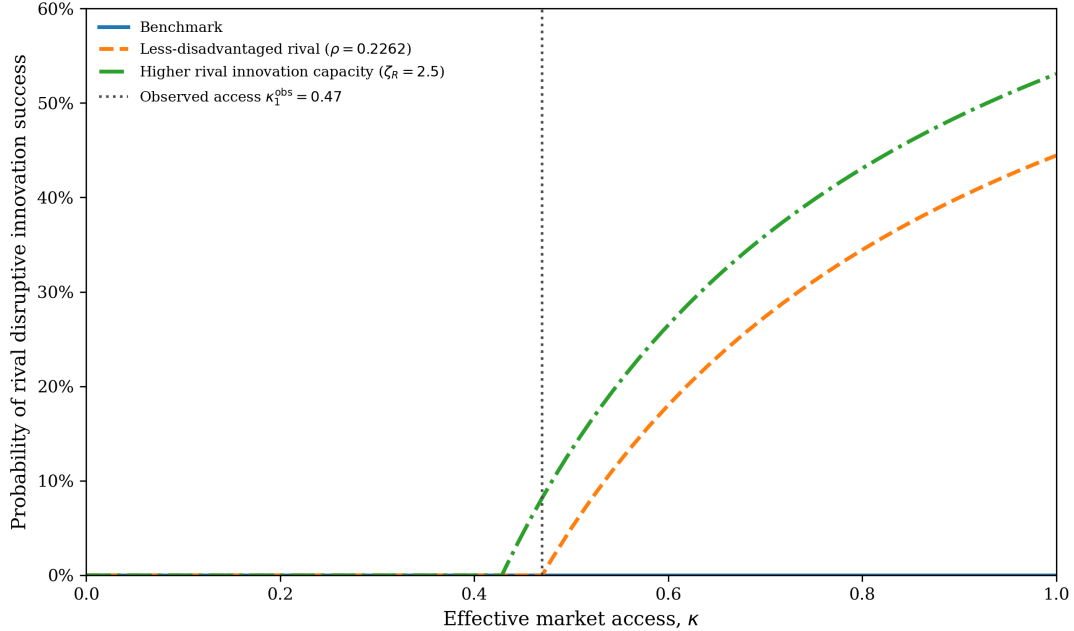


Figure 11: **Rival disruptive innovation across the three paths.** On the benchmark path, rival never innovates. The selected less-disadvantaged rival reaches the innovation boundary at observed access and innovates above it. The higher-capacity rival crosses its innovation threshold before observed access and therefore has a positive success probability at the observed state.

Platform innovation across the three paths. Platform innovation rises along all three paths, even though platform profit falls. To summarize the platform response without imposing

a welfare weight on incremental versus disruptive innovation, Figure 12 plots total platform R&D expenditure, $x_{PI} + x_{PD}$, relative to the common no-improvement level. At observed access, total platform R&D rises by 2.52 percent in the benchmark, 5.11 percent in the selected less-disadvantaged economy, and 2.50 percent under higher rival innovation capacity. At full access, the corresponding increases are 4.91, 11.53, and 6.94 percent.

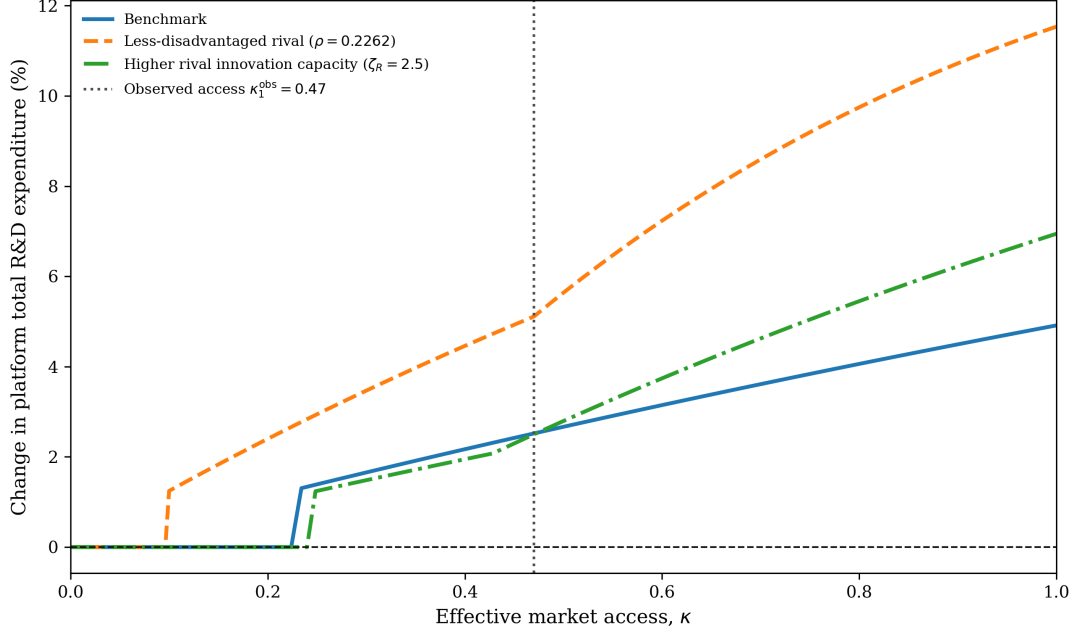


Figure 12: **Platform innovation across the three paths.** The figure reports the percentage change in total platform R&D expenditure, $x_{PI} + x_{PD}$, relative to the common no-improvement level. Platform innovation rises under all three configurations despite declining platform profit.

The platform response is consistent with Proposition 4. Greater market access reduces the platform’s expenditure share and profit, but it also lowers the quality-adjusted digital price, expands digital use, and changes the profit gain from successful innovation. In all three paths, the latter forces are sufficient to raise platform R&D.

Table 8 reports the innovation outcomes at observed and full access.

Table 8: Innovation Across the Three Paths

Outcome	Benchmark		Less-disadvantaged, $\rho = 0.2262$		Higher capacity, $\zeta_R = 2.5$	
	Observed	Full access	Observed	Full access	Observed	Full access
Rival regime	II	II	II/III boundary	III	III	III
Rival share (%)	2.00	4.10	4.29	12.90	2.00	6.30
Platform incremental R&D	0.00903	0.00921	0.00922	0.00968	0.00903	0.00936
Platform disruptive R&D	0.02156	0.02209	0.02214	0.02359	0.02155	0.02255
Rival disruptive R&D	0.00000	0.00000	0.00000	0.00608	0.00035	0.00313
Rival disruptive success (%)	0.00	0.00	0.00	44.44	8.19	53.06
Descriptive total R&D	0.03059	0.03130	0.03136	0.03935	0.03093	0.03504

Notes: “Descriptive total R&D” is the sum of all four investment expenditures. It is a resource-expenditure measure, not a welfare-weighted measure of aggregate innovation output. Incremental and disruptive innovations have different success probabilities and step sizes, and innovations by the two firms need not have equal social value.

Since all innovation expenditures are measured in the same numeraire, it is possible to

plot their sum as those costs. Figure 13 shows that this sum rises by 2.52 percent in the benchmark, 5.11 percent in the selected less-disadvantaged economy, and 3.68 percent under higher innovation capacity at observed access. At full access, the increases are 4.91, 31.90, and 17.43 percent. This aggregation should not be interpreted as “aggregate innovation” in a welfare or technological sense: it does not account for different innovation types, success probabilities, technological step sizes, duplication, or business-stealing effects.

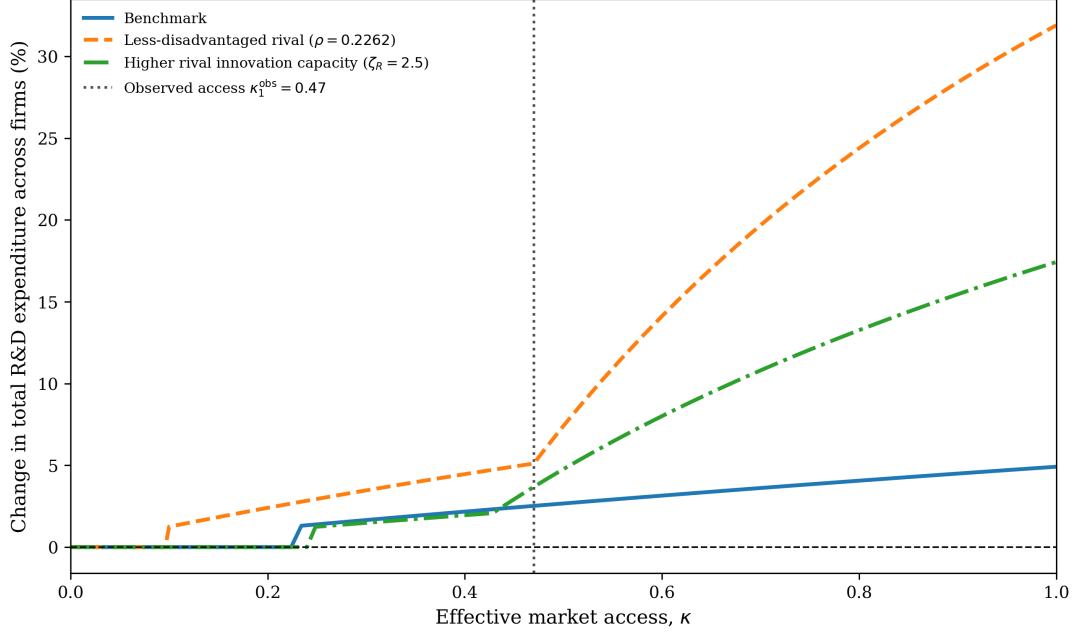


Figure 13: **Descriptive total R&D expenditure across the three paths.** The figure sums platform and rival incremental and disruptive investments and reports the percentage change relative to the common no-improvement level. The sum is informative about resources devoted to R&D but is not a measure of aggregate innovation output or welfare.

Welfare across the three paths. Figure 14 compares total consumption-equivalent welfare gains. Every path is measured relative to its own no-improvement equilibrium at $\kappa_1 = \kappa_0$. We need to note that, for the less-disadvantaged path, the welfare numbers measure the gain from improving market access, given the stronger inherited rival position, rather than the welfare effect of changing ρ itself.

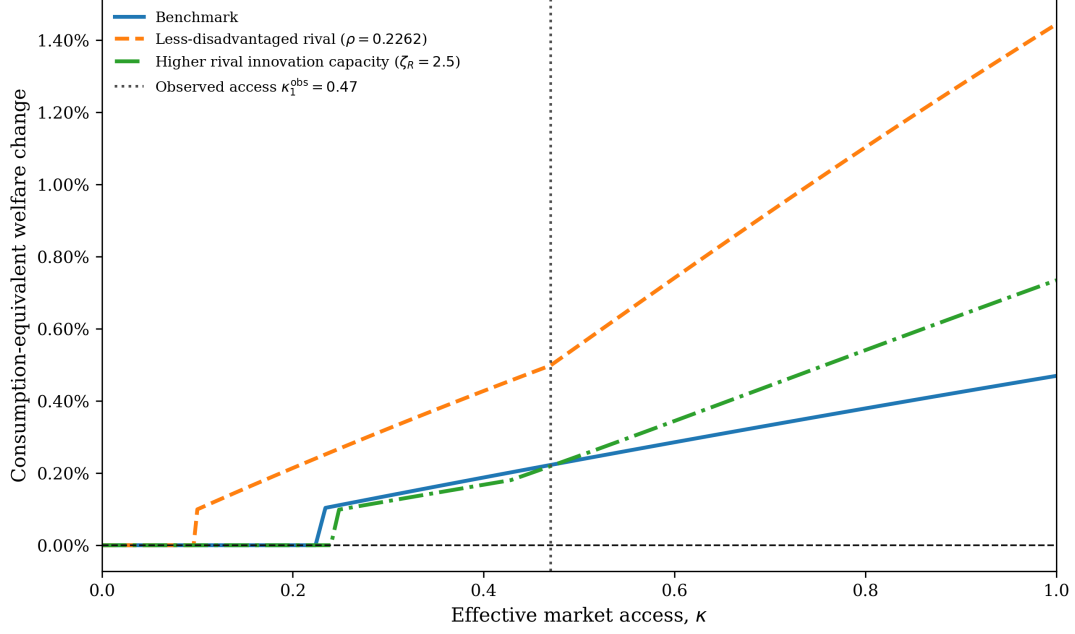


Figure 14: **Welfare effects across the three paths.** Each curve reports the consumption-equivalent welfare change relative to that path’s own no-improvement equilibrium. The less-disadvantaged comparison fixes $\rho = \rho^{I, \text{obs}} = 0.2262$; the higher-capacity comparison sets $\zeta_R = 2.5$ and uses the associated recalibration.

At observed access, the benchmark welfare gain is 0.222%. The selected less-disadvantaged economy produces a larger gain of 0.498% because the same access level supports a larger rival share and a stronger platform response. The higher-capacity economy produces a similar current gain, 0.219%, because it is calibrated to the same 2% rival share and lies only modestly above its innovation threshold.

The paths diverge more strongly at full access. The benchmark welfare gain is 0.470%; the selected less-disadvantaged economy reaches 1.444%; and the higher-capacity economy reaches 0.735%. In the latter two paths, rival disruptive innovation becomes active and amplifies the effects of additional market access.

Table 9 reports the welfare decomposition and associated general equilibrium outcomes.

Table 9: Welfare and General-Equilibrium Outcomes Across the Three Paths

Outcome	Benchmark		Less-disadvantaged, $\rho = 0.2262$		Higher capacity, $\zeta_R = 2.5$	
	Observed	Full access	Observed	Full access	Observed	Full access
CE welfare change (%)	0.222	0.470	0.498	1.444	0.219	0.735
Commercialization/GE component (%)	0.145	0.324	0.348	0.728	0.129	0.290
Innovation component (%)	0.077	0.145	0.150	0.711	0.090	0.443
Wage change (%)	0.443	0.899	0.939	2.728	0.442	1.362
Household-income change (%)	0.484	0.977	1.034	3.601	0.512	1.840
Non-digital-output change (%)	0.315	0.622	0.663	2.370	0.342	1.274
Platform-profit change (%)	-1.623	-3.431	-3.584	-11.587	-1.604	-5.175

Notes: Each change is measured relative to the no-improvement equilibrium within the same path. The welfare decomposition is additive in utility. The displayed consumption-equivalent components need not add exactly to the total because the consumption-equivalent transformation is nonlinear.

The gains propagate beyond direct rival sales. Lower quality-adjusted digital-input prices raise the marginal product of labor in the non-digital sector, increasing wages, household income,

and non-digital output. The strength of this propagation depends materially on the benchmark digital-input share $\iota = 0.30$, which is deliberately high; the magnitudes should therefore be interpreted as results from a reference calibration rather than as a comprehensive benefit–cost estimate of the DMA or the app ecosystem.

Foreign ownership affects domestic incidence rather than the underlying commercialization mechanism. Because 85 percent of platform profit leaves the modeled domestic economy, lower platform profit reduces foreign remittances and allows domestic households to retain a larger share of the gains from lower digital prices and downstream production. From a global perspective, the reduction in remittances is a transfer rather than an additional resource gain.

These three paths show that current market access, inherited quality and productivity, and innovation capacity are complementary. Improved market access creates the commercial base over which entry and innovation earn returns. A less disadvantaged inherited position or greater innovation capacity lowers the access level needed for rival innovation. The model does not causally separate these inherited conditions from the history of market access; it shows how they jointly determine the transition from formal opening to commercialization and dynamic competition.

6.6 Sensitivity and Robustness

Appendix B evaluates 27 valid one-at-a-time alternatives, in addition to the benchmark, and separately reports five operationally nonviable and four no-match calibration boundaries. Unless a target is itself varied, each alternative rematches the inherited household digital-expenditure share, the platform R&D-to-revenue ratio, and the observed rival share; the fixed entry cost is reselected from the interval consistent with inherited non-entry and observed entry. The resulting ranges are comparative-calibration exercises rather than confidence intervals.

The commercial-entry threshold is stable under most alternatives but sensitive to the fixed entry cost. Selecting the tenth and ninetieth percentiles of its identified interval moves κ^E from 0.057 to 0.421, compared with the benchmark value of 0.234. Excluding those direct entry-cost alternatives, valid specifications place the threshold between 0.200 and 0.273. Thus, the existence of a separate entry threshold is stable, while the access improvement required to cross it is weakly identified by the available entry facts.

The benchmark result that rival innovation remains at zero through full market access is not universal. Nine valid alternatives generate an interior innovation threshold, but in every case $\kappa^I > \kappa^E$. The robust implication is that entry happens before innovation, not permanent zero rival innovation. The observed market remains in Regime II in 26 of the 28 valid specifications and moves into Regime III only in the recalibrated $\zeta_R = 2.5$ and $\zeta_R = 3$ exercises. Greater substitution, commercial scale, and rival innovation efficiency are the main forces lowering κ^I .

The positive platform-innovation response is stable across the valid one-at-a-time domain. At observed access, platform incremental investment rises by between 1.15 and 4.19 percent and disruptive investment by between 1.88 and 4.88 percent relative to each alternative’s own no-improvement equilibrium. This is a calibrated robustness result rather than an analytical sign theory.

Every valid alternative produces a positive consumption-equivalent welfare gain. At observed access, the range is 0.063 to 0.277 percent; at full access, it is 0.140 to 0.857 percent. The digital intermediate-input share is the main determinant of the lower end of the range, while substitution and rival innovation capacity govern the upper end. Welfare remains positive when all platform profit accrues domestically, so the result is not generated solely by reduced transfers

to foreign platform owners.

Two additional checks qualify the diagnostic counterfactuals. First, the less-disadvantaged-rival path is driven primarily by the closure of the large inherited relative-demand gap. In other words, closing the benchmark productivity gap alone does not activate rival innovation over the feasible path. Second, varying the observed rival share and measured access value leaves the current market-access term as a minority—between 14.3 and 25.0 percent—of the accounting distance to the 50–50 reference. These exercises preserve the paper’s causal caveat: they describe how inherited conditions enter the model, not why those conditions arose.

7 Discussion and Policy Implications

The European Commission’s first review of the DMA provides a direct policy context for the mechanisms studied in this paper. The review finds that the DMA has already created new opportunities for business users and alternative providers. But the effectiveness of these opportunities depends on practical implementation and continued enforcement. In app-distribution, the Commission identifies fees, multi-step installation, vetting procedures, and other technical or procedural frictions as factors preventing alternative distribution channels from being meaningfully usable in practice. (European Commission, 2026b,a).

The evidence closely matches what we derived from the model and quantitative analysis. Formal opening does not itself determine effective market access, and effective market access does not guarantee rival commercially viable entry. Moreover, the effective market access required to support entry may be lower than that required to support commercial-scale and rival innovation.

The model isolates one foundational channel through which digital market regulation can improve contestability—whether a rival can use a route controlled by a gatekeeper to reach users and build a commercial base. The framework does not rank any individual obligations, or estimate enforcement resources to produce a given value of κ , or identify the historical cause of the inherited rival position. The policy implications should be interpreted as the implications for the regulations or obligations that aim at unlocking the bottleneck, rather than a complete assessment of those regulations.

When does improved market access produce commercially viable entry? The first implication is that commercially viable entry is different from usable market access and formal legal opening. A legal obligation might create a right for the rivals to use a route to market, but it might not make the route sufficiently practical or attractive for users. So even if the market access improves, the rival enters only when the expected commercial opportunity can cover its fixed entry cost.

The commercial-entry threshold also implies that market access reforms are state-dependent. A large improvement that starts from a very low inherited competitive state may leave the market still below κ^E and fail to generate rival entry, even though it increases the demand that a rival could obtain conditional on its entry. However, a small improvement from just below the threshold can trigger a discrete change in market structure. Once the threshold is crossed, additional market access can help an active rival expand its sales base.

Policy should use realized effective market access rather than the formal strictness of the legal rule as its objective. Default placement, installation procedures, technical interfaces, interoperability, and switching costs may all affect whether a legally available channel can generate user adoption and usage. The model does not show how a particular obligation or

enforcement action changes effective market access. It shows why regulations should ultimately be evaluated through the commercial opportunities that new entrants can actually use.

When does entry become dynamic competition? The second implication is that a rival can enter but still lacks sufficient commercial scale to invest in innovation. Entry demonstrates that the rival has acquired a commercial foothold, but it does not establish that the foothold is large or profitable enough to support continued improvement in quality and productivity.

In the benchmark calibration, the economy remains in Regime II (rival entry without innovation) throughout the full period of market access. However, the complementary exercises show that this result is conditional on the inherited competitive state. A stronger inherited rival quality and productivity position or greater rival innovation capacity lowers the effective market access level required for rival innovation.

The inherited competitive state reflects product gaps between the platform and the rival. But it should not be interpreted as an intrinsic measure of rival merit, since it may also reflect the effects of networks, accumulated data, reputation, complementary services, user familiarity, and prior opportunities. These conditions can be partially attributed to earlier market access status. So, a small rival market share after the regulation does not necessarily show that the remaining competitive gap is attributable to insufficient rival effort.

Moreover, the model holds the inherited state as given when the market access changes. Continuous market access may gradually affect that state through user adoption and accumulated data and reputation. These dynamic feedback are not looped back to this two-period model. The results actually show that current market access and inherited rival capacity are complementary in determining the transition of rival entry and innovation.

Policy evaluation should move beyond counting entrants or observing an initial market share. It should also examine whether rivals retain users, generate recurring revenue, and acquire the capacity to invest.

How does market opening affect platform R&D and welfare? The third implication is that a decline in incumbent profit is not evidence that incumbent innovation incentives have weakened. In other words, preserving incumbent rent is not a necessary condition for maintaining its R&D.

The effect of greater market access on platform innovation is not theoretically predetermined. Higher market access allows rivals to enter into the market and reduces the platform's market share and profit, while digital market expansion and changes in the payoff from successful innovation can strengthen platform's R&D incentives. The model does not imply a clear effect of opening up a market on platform innovation.

In the benchmark economy, platform total R&D expenditure increases although its profit falls with leveled market access. The positive platform investment in innovation is universal across all sensitivity exercises. Policy assessment should distinguish platform profit from its R&D expenditure and the value of the resulting improvements.

The welfare effects also extend beyond the rival's direct market share. Rival entry and commercialization lower the quality-adjusted digital price. Since digital services enter non-digital production as intermediate inputs, this price change affects labor productivity, wages, household income, and non-digital outputs. Endogenous innovation adds an additional channel.

The benchmark consumption-equivalent gain is 0.22 percent at observed market access and 0.47 percent at full market access. The gain is larger in economies in which rival innovation becomes active. The magnitude of the welfare effects are sensitive to the assumed importance

of digital inputs in downstream non-digital production and to domestic ownership of platform profits. Nevertheless, the qualitative implication is robust: the economic importance of a digital market bottleneck may exceed the direct household expenditure share of the immediate service, but the magnitude of that propagation requires market-specific empirical evidence.

Last but not the least. The framework imposes a disciplined sequence on the evaluation of bottleneck-opening regulation: formal opening $\longrightarrow$ usable market access $\longrightarrow$ commercial entry $\longrightarrow$ rival innovation.

A comprehensive evaluation of digital-market regulation requires distinct evidence across multiple stages. Formal compliance signals the creation of legal frameworks, whereas effective market access demonstrates whether competitors can leverage these frameworks in practice. Furthermore, long-term commercial viability and expansion indicate whether a truly dynamic competitive process has emerged. Isolating these distinct outcomes provides a richer, more nuanced analysis than relying solely on compliance metrics, entrant numbers, or snapshot market shares.

8 Conclusion

This paper investigates when opening digital market bottlenecks creates genuine competition rather than a mere formal legal opportunity. Answering this requires distinguishing between three outcomes often conflated as equivalent: rival commercially viable entry, commercialization, and sustained innovation. I formalize this sequence using a two-period stochastic general-equilibrium model. In this model, effective market access expands the demand available to a potential rival, whereas fixed entry costs establish a commercially viable entry threshold. Conditional on crossing this entry hurdle, scale-dependent innovation returns create a second, higher threshold for rival innovation. These two thresholds partition the market into three distinct regimes: no rival entry, rival entry without innovation, and rival entry with innovation. Crucially, their boundaries depend on current effective market access interacting with the inherited competitive state, entry costs, and innovation capacity.

Applying the model to EU mobile app distribution illustrates the distinction between commercial entry and dynamic competition. The observed post-DMA effective market access $\kappa_1^{\text{obs}} = 0.47$ crosses the commercial entry threshold $\kappa^E = 0.2342$ and supports rival entry, yet the rival captures only 2% of the market share. Even at full effective market access $\kappa = 1$, the rival's market share reaches only 4.1%, remaining trapped in Regime II where the marginal return to disruptive innovation fall short of the investment boundary. However, complementary exercises demonstrate that this outcome is not an inherent limit of access regulation. Narrowing the rival's quality and productivity gaps by 22.62% lowers the innovation threshold to observed market access level, even with a merely 4.29% market share. A higher-innovation-capacity economy reaches Regime III at $\kappa^I = 0.4284$. Dynamic viability does not require market share parity and markets with similar entry profiles can have very different innovation prospects.

Greater market access also reshapes platform strategy. In the benchmark calibration, platform profit falls while platform innovation investments rise. Overall welfare improves through rival commercialization, innovation, lower quality-adjusted prices, and downstream linkages, yielding consumption-equivalent gains of 0.222% at observed access and 0.470% at full access (rising to 0.735% under higher-innovation-capacity). Sensitivity analysis confirms that the ordering of rival entry before innovation and the positive welfare effects are robust across valid parameter domains.

This framework provides a theory-first methodology to evaluate bottleneck-opening regulations in digital market, showing that formal compliance, usability, entry, market share, innovation, and welfare are distinct, sequential indicators. Future work can extend this two-period structure to dynamic settings with endogenous state variables to analyze how persistent access gradually unwinds historical platform advantages. Ultimately, a market is commercially open when a rival can enter and survive; it becomes dynamically contestable only when that commercial footprint supports continuous innovation. The gap between these two outcomes defines the economic space where implementation, market structure, and innovation policy interact.

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Appendix A Existence of Equilibrium

This appendix proves the existence of equilibrium. The argument proceeds in four steps. First, I construct the general-equilibrium allocation conditional on rival entry and realized innovation outcomes. Second, I show that firms' innovation choices can be restricted to compact sets. Third, I establish the existence of a pure-strategy equilibrium in the innovation subgame. Finally, I incorporate the rival's entry decision.

A.1 The State-Contingent General Equilibrium

Fix a period, the rival's entry decision, effective contestability, and a realization of all innovation outcomes. Consequently, the qualities q_j , productivities A_j , and quality-adjusted prices $p_j = P_j/q_j$ are fixed within the state.

If the rival is active, its within-digital expenditure share is

$$s_R = \frac{\kappa^{\sigma_D} p_R^{1-\sigma_D}}{p_P^{1-\sigma_D} + \kappa^{\sigma_D} p_R^{1-\sigma_D}}, \quad s_P = 1 - s_R. \quad (\text{A.1})$$

If the rival is inactive, I set $s_R = 0$ and $s_P = 1$. Let P_D denote the corresponding digital-service price index, and let $S^H \in (0, 1)$ denote the household's total digital expenditure share, as derived in the household problem.

A.1.1 Non-digital production and the wage

The non-digital producer's total cost of producing Y_N is

$$\mathcal{C}_N = \min_{L_N, \tilde{D}} \left\{ wL_N + P_D \tilde{D} \right\} \quad \text{subject to} \quad BL_N^{1-\iota} \tilde{D}^\iota \geq Y_N. \quad (\text{A.2})$$

Because the production function has constant returns to scale, the total minimized cost is proportional to output:

$$\mathcal{C}_N = c_N Y_N, \quad (\text{A.3})$$

where unit cost is the total minimized cost divided by output,

$$c_N \equiv \frac{\mathcal{C}_N}{Y_N} = \frac{1}{B} \left(\frac{w}{1-\iota} \right)^{1-\iota} \left(\frac{P_D}{\iota} \right)^\iota. \quad (\text{A.4})$$

Competition in the non-digital sector and the normalization of the non-digital-good price to one imply

$$c_N = 1. \quad (\text{A.5})$$

The equilibrium wage is therefore uniquely determined by

$$w = (1-\iota)B^{\frac{1}{1-\iota}} \left(\frac{\iota}{P_D} \right)^{\frac{\iota}{1-\iota}} > 0. \quad (\text{A.6})$$

In a competitive market, non-digital firms run at zero profit, and the associated conditional factor demands satisfy

$$wL_N = (1-\iota)Y_N, \quad P_D \tilde{D} = \iota Y_N. \quad (\text{A.7})$$

Implication for Equilibrium Existence Equation (A.6) pins down a unique wage.

A.1.2 Digital revenue, labor demand, and profits

Total expenditure on digital services equals household digital expenditure plus the non-digital sector's expenditure on digital intermediate inputs:

$$E^D = S^H I + \iota Y_N, \quad (\text{A.8})$$

where E^D denotes total expenditure on digital services, S^H denotes the share of household expenditure on digital services, and I denotes household income.

Because households and the non-digital producer have the same within-digital aggregator, supplier revenues are

$$P_P Y_P = s_P E^D, \quad P_R Y_R = s_R E^D. \quad (\text{A.9})$$

Using the digital production technology $Y_j = A_j L_j$, labor demand by each supplier is

$$L_P = \frac{s_P E^D}{P_P A_P}, \quad L_R = \frac{s_R E^D}{P_R A_R}. \quad (\text{A.10})$$

Total labor demanded by the digital sector is

$$L^D = L_P + L_R = \frac{s_P E^D}{P_P A_P} + \frac{s_R E^D}{P_R A_R} \quad (\text{A.11})$$

Define labor required per unit of digital expenditure as

$$\ell \equiv \frac{s_P}{P_P A_P} + \frac{s_R}{P_R A_R}. \quad (\text{A.12})$$

Then the aggregate digital labor demand can be expressed as

$$L^D = \ell E^D. \quad (\text{A.13})$$

For an active supplier, define its unit operating-surplus share as

$$\nu_j \equiv 1 - \frac{w}{P_j A_j}. \quad (\text{A.14})$$

The operational-viability condition in Proposition ?? requires

$$\nu_j > 0 \quad \text{for every active supplier.} \quad (\text{A.15})$$

Under this condition, an active supplier serves all the demand allocated to it. Operating profits are

$$\pi_P = s_P \nu_P E^D, \quad \pi_R = s_R \nu_R E^D. \quad (\text{A.16})$$

Implication for Equilibrium Existence The positive-operating-profit condition (A.15) guarantees that each active supplier optimally and uniquely serves all demand allocated to it. It therefore makes the demand system consistent with the firms' conditional production decisions. It is not a restriction requiring net profit after entry and innovation costs to be positive in every state.

A.1.3 Resources feasibility and household income

The net transfer to the platform's foreign owners is

$$T_t^F = \underbrace{\phi\pi_{Pt}}_{\text{foreign share of operating profit}} - \underbrace{1\{t=0\}\phi\mathcal{C}_P(\mathbf{x}_P)}_{\text{foreign financing of platform innovation}}, \quad (\text{A.17})$$

where $\phi \in [0, 1]$ denotes the foreign ownership share of the dominant platform. Thus, $T_0^F = \phi[\pi_{P0} - \mathcal{C}_P(\mathbf{x}_P)]$, whereas $T_1^F = \phi\pi_{P1}$.

The full non-digital resource constraint is

$$Y_{Nt} = \underbrace{C_t}_{\text{household non-digital consumption}} + \underbrace{1\{t=0\}[\mathcal{C}_P(\mathbf{x}_P) + e_R(F_R + \mathcal{C}_R(\mathbf{x}_R))]}_{\text{entry and innovation resource use}} + \underbrace{T_t^F}_{\text{net transfer to foreign owners}}. \quad (\text{A.18})$$

Let $\mathcal{K}_t^D$ denote the domestic-financed component of transition expenditures:

$$\mathcal{K}_t^D = \begin{cases} (1 - \phi)\mathcal{C}_P(\mathbf{x}_P) + e_R[F_R + \mathcal{C}_R(\mathbf{x}_R)], & t = 0, \\ 0, & t = 1. \end{cases} \quad (\text{A.19})$$

Denote the fraction of total digital expenditure remitted abroad through platform profits as f and it can be expressed as

$$f \equiv \phi s_P \nu_P. \quad (\text{A.20})$$

where $\nu_P \in (0, 1)$ is the unit operating profit share of the platform, and $s_P \in [0, 1]$ is the digital expenditure share of the platform. We can see that $f < 1$.

Combining the non-digital resource constraint with foreign profit leakage gives

$$Y_{Nt} = \underbrace{(1 - S_t^H)I_t}_{\text{household non-digital consumption}} + \underbrace{\mathcal{K}_t^D}_{\text{domestically financed entry and innovation}} + \underbrace{f_t E_t^D}_{\text{foreign platform-profit leakage}}. \quad (\text{A.21})$$

Substituting equation (A.8) into equation (A.21) yields

$$(1 - f_\iota)Y_{Nt} = [1 - (1 - f)S_t^H] I_t + \mathcal{K}_t^D. \quad (\text{A.22})$$

Hence,

$$Y_{Nt} = hI_t + k_t, \quad (\text{A.23})$$

where

$$h \equiv \frac{1 - (1 - f)S_t^H}{1 - f_\iota} > 0, \quad k \equiv \frac{\mathcal{K}_t^D}{1 - f_\iota} \geq 0. \quad (\text{A.24})$$

Both denominators are strictly positive because $f < 1$ and $\iota \in (0, 1)$.

Household income identity I in each period can be expanded as

$$I_0 = \underbrace{w_0 \bar{L}}_{\text{labor income}} + \underbrace{(1 - \phi)\pi_{P0} + \pi_{R0}}_{\text{domestically accruing operating profit}} - \underbrace{\left[(1 - \phi)\mathcal{C}_P(\mathbf{x}_P) + e_R(F_R + \mathcal{C}_R(\mathbf{x}_R)) \right]}_{\text{domestically financed transition expenditures}}, \quad (\text{A.25})$$

$$I_1 = \underbrace{w_1 \bar{L}}_{\text{labor income}} + \underbrace{(1 - \phi)\pi_{P1} + \pi_{R1}}_{\text{domestically accruing operating profit}}. \quad (\text{A.26})$$

A.1.4 Labor-market clearing

Total labor demand is the sum of non-digital and digital labor demand. Using equations (A.7) and (A.13), labor-market clearing requires

$$\frac{1 - \iota}{w} Y_N + \ell (S^H I + \iota Y_N) = \bar{L}. \quad (\text{A.27})$$

Define

$$a \equiv \frac{1 - \iota}{w} + \iota \ell > 0. \quad (\text{A.28})$$

Substituting $Y_N = hI + k$ into labor-market clearing gives

$$(ah + \ell S^H) I + ak = \bar{L}. \quad (\text{A.29})$$

Household income is therefore uniquely determined:

$$I = \frac{\bar{L} - ak}{ah + \ell S^H}. \quad (\text{A.30})$$

The denominator is strictly positive. Positive equilibrium income requires

$$\boxed{\bar{L} > ak} \iff \bar{L} > \left(\frac{1 - \iota}{w} + \iota \ell \right) \frac{\mathcal{K}_t^D}{1 - f_\iota}. \quad (\text{A.31})$$

Equation (A.31) is the transition-feasibility condition referenced in Proposition ???. It states that entry and innovation investments cannot exceed the resources the economy can finance during the transition.

Given I , equation (A.23) uniquely determines Y_N . Equations (A.8)–(A.10) then uniquely determine digital expenditures, outputs, labor demands, and profits.

The household income identity is also satisfied. In particular,

$$\begin{aligned} w\bar{L} + (1 - \phi)\pi_P + \pi_R - \mathcal{K}_t^D &= (1 - \iota)Y_N + (1 - f)E^D - \mathcal{K}_t^D \\ &= Y_N + S^H I - fE^D - \mathcal{K}_t^D \\ &= I, \end{aligned} \quad (\text{A.32})$$

To see (A.32), we have

$$\begin{aligned} w_t L_t^D &= \sum_{j \in \{P, R\}} \frac{w_t}{P_j A_{jt}} s_{jt} E_t^D \\ &= \sum_{j \in \{P, R\}} s_{jt} (1 - \nu_{jt}) E_t^D. \end{aligned} \quad (\text{A.33})$$

$$\pi_{Pt} + \pi_{Rt} = \sum_{j \in \{P, R\}} s_{jt} \nu_{jt} E_t^D. \quad (\text{A.34})$$

$$w_t L_t^D + \pi_{Pt} + \pi_{Rt} = E_t^D. \quad (\text{A.35})$$

$$w_t L_{Nt} = (1 - \iota) Y_{Nt}. \quad (\text{A.36})$$

$$\begin{aligned} w_t \bar{L} + \pi_{Pt} + \pi_{Rt} &= w_t L_{Nt} + w_t L_t^D + \pi_{Pt} + \pi_{Rt} \\ &= (1 - \iota) Y_{Nt} + E_t^D. \end{aligned} \quad (\text{A.37})$$

$$\begin{aligned} I_t &= (1 - \iota) Y_{Nt} + E_t^D - \underbrace{\phi \pi_{Pt}}_{f_t E_t^D} - \mathcal{K}_t^D \\ &= (1 - \iota) Y_{Nt} + (1 - f_t) E_t^D - \mathcal{K}_t^D. \end{aligned} \quad (\text{A.38})$$

Thus, the household budget, firm profits, resource constraint, and labor-market clearing are mutually consistent. We have established the following intermediate result.

Lemma 1 (State-contingent equilibrium). *For any fixed entry decision and innovation realization, operational viability and transition feasibility imply that the state-contingent general equilibrium exists and is unique.*

A.2 Compact Innovation Strategy Sets

Innovation effort affects the probabilities of innovation outcomes but not the size of the technology improvement, conditional on success. Since each firm has two binary innovation outcomes for each type of innovation,

$$\boldsymbol{\xi}_j = (\xi_{jI}, \xi_{jD}) \in \{0, 1\}^2. \quad (\text{A.39})$$

thus, each firm has four possible own innovation-outcome combinations. When both firms are active, the joint innovation state is

$$\boldsymbol{\xi} = (\xi_{PI}, \xi_{PD}, \xi_{RI}, \xi_{RD}) \in \{0, 1\}^4, \quad (\text{A.40})$$

Therefore, there are $2^4 = 16$ joint innovation states. If the rival does not enter, only the platform's four innovation states remain. Because the innovation states are finite, the set of possible productivity and quality states is finite.

Moreover, digital labor cannot exceed the aggregate labor endowment. Hence, state-contingent operating profit is bounded above by

$$0 \leq \pi_{j1}(\boldsymbol{\xi}) \leq P_j A_{j1}(\boldsymbol{\xi}) \bar{L}. \quad (\text{A.41})$$

Define

$$\bar{\pi}_{j1} \equiv \max_{\boldsymbol{\xi}} \{P_j A_{j1}(\boldsymbol{\xi}) \bar{L}\} < \infty. \quad (\text{A.42})$$

Up to payoff components independent of its innovation choices, firm j 's innovation payoff is

$$U_j(\mathbf{x}_j, \mathbf{x}_{-j}) = -\mathcal{C}_j(\mathbf{x}_j) + \beta \mathbb{E}[\pi_{j1}(\boldsymbol{\xi}) \mid \mathbf{x}_j, \mathbf{x}_{-j}]. \quad (\text{A.43})$$

where $\mathcal{C}_j(\mathbf{x}_j) = x_{jI} + x_{jD}$.

Any strategy satisfying $\mathcal{C}_j(\mathbf{x}_j) > \beta \bar{\pi}_j$ yields a strictly lower payoff than a zero-investment strategy. We are able to restrict firm j 's innovation choices to

$$\mathbb{X}_j^{\text{inv}} = \{(x_{jI}, x_{jD}) \in \mathbb{R}_+^2 : x_{jI} + x_{jD} \leq \beta \bar{\pi}_j\}. \quad (\text{A.44})$$

This set is nonempty because it contains $(0, 0)$. It is closed because it is defined by weak inequalities, and it is bounded because each component satisfies $0 \leq x_{jk} \leq \beta \bar{\pi}_j$. It is therefore compact.

The set is also convex. For any $\mathbf{x}_j, \mathbf{y}_j \in \mathbb{X}_j^{\text{inv}}$ and any $\alpha \in [0, 1]$,

$$\begin{aligned} \mathcal{C}_j(\alpha \mathbf{x}_j + (1 - \alpha) \mathbf{y}_j) &= \alpha \mathcal{C}_j(\mathbf{x}_j) + (1 - \alpha) \mathcal{C}_j(\mathbf{y}_j) \\ &\leq \beta \bar{\pi}_j, \end{aligned} \quad (\text{A.45})$$

where the equality follows from the linear innovation-cost function $\mathcal{C}_j(\mathbf{x}_j) = x_{jI} + x_{jD}$. Hence, $\alpha \mathbf{x}_j + (1 - \alpha) \mathbf{y}_j \in \mathbb{X}_j^{\text{inv}}$.

Therefore, in sum, the set $\mathbb{X}_j^{\text{inv}}$ is nonempty, compact, and convex. Transition feasibility is assumed to hold over these payoff-relevant strategy sets.

Implication for Equilibrium Existence Bounded operating profits and linear innovation costs allow each firm's unbounded original innovation domain to be replaced by a nonempty, compact, and convex innovation choice set without eliminating any optimal strategy. This provides the compact-strategy-set condition needed for the existence of the innovation equilibrium.

A.3 The Strategic Innovation Subgame

The innovation-success probabilities

$$\theta_{jk}(x_{jk}) = 1 - \exp(-\eta a_{jk} x_{jk}), \quad (\text{A.46})$$

are continuous in investment. Since the number of innovation states is finite, and state-contingent equilibrium profits are well-defined by Lemma 1, expected payoffs are continuous in both firms' innovation choices.

To characterize a sufficient curvature condition, we can fix the opponent's strategy and integrate over the opponent's innovation outcomes. Define

$$\bar{\pi}_j^{ab} \equiv \mathbb{E}_{\boldsymbol{\xi}_{-j}}[\pi_{j1} \mid \xi_{jI} = a, \xi_{jD} = b], \quad a, b \in \{0, 1\}. \quad (\text{A.47})$$

Define the incremental gain, the disruptive gain as

$$\Delta_j^I \equiv \bar{\pi}_j^{10} - \bar{\pi}_j^{00}, \quad (\text{A.48})$$

$$\Delta_j^D \equiv \bar{\pi}_j^{01} - \bar{\pi}_j^{00}, \quad (\text{A.49})$$

$$(\text{A.50})$$

The interaction term of the two is a difference-in-differences:

$$\Delta_j^{ID} = \underbrace{(\bar{\pi}_j^{11} - \bar{\pi}_j^{01})}_{\text{effect of incremental success when disruptive innovation succeeds}} - \underbrace{(\bar{\pi}_j^{10} - \bar{\pi}_j^{00})}_{\text{effect of incremental success when disruptive innovation fails}}. \quad (\text{A.51})$$

Equivalently,

$$\Delta_j^{ID} = \underbrace{(\bar{\pi}_j^{11} - \bar{\pi}_j^{10})}_{\text{effect of disruptive success when incremental innovation succeeds}} - \underbrace{(\bar{\pi}_j^{01} - \bar{\pi}_j^{00})}_{\text{effect of disruptive success when incremental innovation fails}}. \quad (\text{A.52})$$

The interaction term Δ_j^{ID} measures whether the profit effect of one innovation depends on the success of the other. It is positive when the two innovation outcomes are complementary in generating profits, zero when their profit effects are additive, and negative when they are substitutes. In other words, the interaction term captures complementarity or substitutability between the two innovation directions.

- $\Delta_j^{ID} > 0$: the two successes are complementary in profits.
- $\Delta_j^{ID} = 0$: their profit effects are additive.
- $\Delta_j^{ID} < 0$: the two successes are substitutes or partially redundant.

We can use Δ_j^I , Δ_j^D , and Δ_j^{ID} to reconstruct every innovation outcome:

$$\begin{aligned} \bar{\pi}_j^{10} &= \bar{\pi}_j^{00} + \Delta_j^I, \\ \bar{\pi}_j^{01} &= \bar{\pi}_j^{00} + \Delta_j^D, \\ \bar{\pi}_j^{11} &= \bar{\pi}_j^{00} + \Delta_j^I + \Delta_j^D + \Delta_j^{ID}. \end{aligned} \quad (\text{A.53})$$

Given the innovation success probability θ_{jk} , assuming the two outcomes are conditionally independent, the expected continuation profit is initially

$$\begin{aligned} \mathbb{E}_0[\pi_{j1}] &= (1 - \theta_{jI})(1 - \theta_{jD})\bar{\pi}_j^{00} \\ &\quad + \theta_{jI}\theta_{jD}\bar{\pi}_j^{11} \\ &\quad + \theta_{jI}(1 - \theta_{jD})\bar{\pi}_j^{10} \\ &\quad + (1 - \theta_{jI})\theta_{jD}\bar{\pi}_j^{01}, \end{aligned} \quad (\text{A.54})$$

Substituting equation (A.53), the expected continuation profit can then be written as

$$\mathbb{E}_0[\pi_{j1}] = \bar{\pi}_j^{00} + \Delta_j^I\theta_{jI} + \Delta_j^D\theta_{jD} + \Delta_j^{ID}\theta_{jI}\theta_{jD}. \quad (\text{A.55})$$

To establish a sufficient condition for payoff concavity, define the marginal profit gains associated with the two success probabilities as

$$M_{jI} \equiv \frac{\partial G_j}{\partial \theta_{jI}} = \Delta_j^I + \Delta_j^{ID}\theta_{jD}, \quad (\text{A.56})$$

$$M_{jD} \equiv \frac{\partial G_j}{\partial \theta_{jD}} = \Delta_j^D + \Delta_j^{ID}\theta_{jI}. \quad (\text{A.57})$$

These terms have a direct economic interpretation. In particular,

$$M_{jI} = (1 - \theta_{jD}) (\pi_j^{10} - \pi_j^{00}) + \theta_{jD} (\pi_j^{11} - \pi_j^{01}), \quad (\text{A.58})$$

$$M_{jD} = (1 - \theta_{jI}) (\pi_j^{01} - \pi_j^{00}) + \theta_{jI} (\pi_j^{11} - \pi_j^{10}). \quad (\text{A.59})$$

Thus, M_{jI} is the expected profit gain from incremental success, averaging over disruptive innovation outcomes, while M_{jD} is the corresponding expected gain from disruptive success.

Let $b_{jk} \equiv \eta a_{jk}$. The first and the second derivative of innovation success probability are

$$\theta'_{jk} = b_{jk}(1 - \theta_{jk}) > 0, \quad \theta''_{jk} = -b_{jk}^2(1 - \theta_{jk}) < 0. \quad (\text{A.60})$$

Let $G_j = \mathbb{E}_0 [\pi_{j1}]$. The first derivatives of G_j are

$$\begin{aligned} \frac{\partial G_j}{\partial x_{jI}} &= \Delta_j^I \theta'_{jI} + \Delta_j^{ID} \theta_{jD} \theta'_{jI} \\ &= (\Delta_j^I + \Delta_j^{ID} \theta_{jD}) \theta'_{jI} \\ &= b_{jI} (1 - \theta_{jI}) M_{jI}, \end{aligned} \quad (\text{A.61})$$

and

$$\begin{aligned} \frac{\partial G_j}{\partial x_{jD}} &= \Delta_j^D \theta'_{jD} + \Delta_j^{ID} \theta_{jI} \theta'_{jD} \\ &= (\Delta_j^D + \Delta_j^{ID} \theta_{jI}) \theta'_{jD} \\ &= b_{jD} (1 - \theta_{jD}) M_{jD}. \end{aligned} \quad (\text{A.62})$$

The Hessian of expected continuation profit is

$$\nabla^2 G_j = \begin{pmatrix} -b_{jI}^2(1 - \theta_{jI})M_{jI} & \Delta_j^{ID} b_{jI} b_{jD} (1 - \theta_{jI})(1 - \theta_{jD}) \\ \Delta_j^{ID} b_{jI} b_{jD} (1 - \theta_{jI})(1 - \theta_{jD}) & -b_{jD}^2(1 - \theta_{jD})M_{jD} \end{pmatrix}. \quad (\text{A.63})$$

A symmetric 2×2 matrix is negative semidefinite when its diagonal elements are nonpositive and its determinant is nonnegative. The diagonal elements in equation (A.63) are nonpositive if

$$M_{jI} \geq 0, \quad M_{jD} \geq 0. \quad (\text{A.64})$$

Moreover,

$$\begin{aligned} \det(\nabla^2 G_j) &= b_{jI}^2 b_{jD}^2 (1 - \theta_{jI})(1 - \theta_{jD}) \\ &\quad \times [M_{jI} M_{jD} - (\Delta_j^{ID})^2 (1 - \theta_{jI})(1 - \theta_{jD})]. \end{aligned} \quad (\text{A.65})$$

The determinant is therefore nonnegative if

$$M_{jI} M_{jD} \geq (\Delta_j^{ID})^2 (1 - \theta_{jI})(1 - \theta_{jD}). \quad (\text{A.66})$$

Under equations (A.64) and (A.66), the Hessian is negative semidefinite, so expected continuation profit is concave in the firm's own innovation choices. Because innovation cost is linear, subtracting $\mathcal{C}_j(\mathbf{x}_j)$ does not alter this curvature. The full innovation payoff is also concave and hence quasi-concave.

Implication for Equilibrium Existence The compactness and convexity of $\mathbb{X}_j^{\text{inv}}$, continuity of payoffs, and quasi-concavity in each firm’s own strategy imply, by the Debreu–Fan–Glicksberg theorem, that the innovation subgame has a pure-strategy Nash equilibrium.

A.4 Entry and Completion of the Proof

Consider first the continuation subgame following rival entry. The preceding argument has established the existence of an innovation equilibrium $(\mathbf{x}_P^*, \mathbf{x}_R^*)$. If the rival does not enter, its innovation choices and production are zero, and the platform’s innovation problem has a solution on its compact strategy set.

The rival’s value from entry is

$$\mathcal{V}_R^E = -F_R - \mathcal{C}_R(\mathbf{x}_R^*) + \beta \mathbb{E}[\pi_{R1}^* \mid \mathbf{x}_P^*, \mathbf{x}_R^*]. \quad (\text{A.67})$$

The no-entry value is normalized to zero. Because the entry set $\{0, 1\}$ is finite, an optimal entry decision exists:

$$e_R^* = \begin{cases} 1, & \mathcal{V}_R^E \geq 0, \\ 0, & \mathcal{V}_R^E < 0. \end{cases} \quad (\text{A.68})$$

Implication for Equilibrium Existence Because entry is a choice from the finite set $\{0, 1\}$ and an equilibrium exists in each continuation innovation subgame, an optimal entry decision exists. Combining this decision with the corresponding innovation equilibrium and state-contingent market allocation completes the proof of Proposition ??.

Appendix B Sensitivity Analysis

From the sensitivity analysis exercises, we learned five things. First, a rival-commercial-entry threshold exists throughout the valid domain of effective market access, though its exact location is most sensitive to the fixed entry cost. Second, rival-post-entry commercialization is more sensitive to within-digital substitution and observed access–share combination. Third, rival entry precedes rival innovation in every valid specification with an interior innovation threshold. And the platform increases its innovation in both directions. Fourth, welfare remains positive across all valid alternatives, but its magnitude depends on downstream digital-input use, domestic ownership, and rival market position.

Fifth, the less-disadvantaged-rival path is driven primarily by the large inherited relative-demand gap in the benchmark, while the exact share-gap accounting split remains sensitive to the observed access and market-share moments.

B.1 Sensitivity Analysis Design

I examine the robustness of the quantitative results to alternative parameter values and calibration moments. Each exercise changes one assigned parameter or one target moment at a time. I then recalibrate the remaining parameters that are determined internally. Each alternative continues to match three moments: i) the inherited household digital expenditure share; ii) the platform’s R&D to revenue ratio; and iii) the rival’s 2 percent expenditure share at the observed effective market access.

Recalibration proceeds with four steps. First, I choose an upper-tier CES weight μ to match the household digital expenditure share, and adjust non-digital sector productivity B to preserve the inherited wage normalization. Second, I calibrate the common innovation-efficiency parameter η to match the platform R&D moment. Third, to match the observed rival share, I calibrate the inherited rival quality q_{R0} . Finally, I recompute the range of fixed entry costs F_R .¹⁹ These exercises aim at testing the location of the commercial-entry threshold, the rival-innovation threshold, and the magnitude of welfare effects.

Table B.1 presents different types of specification in these exercises.

Table B.1: Classification of Sensitivity Specifications

Classification	Definition and required conditions	Treatment in the analysis
<i>Valid</i>	The specification needs to: (i) reproduces the required calibration moments; (ii) preserves inherited non-entry and observed post-DMA entry; (iii) converges to firms' mutual best responses; (iv) clears all markets; (v) satisfies transition feasibility; and (vi) delivers strictly positive operating profit in every positive-probability state at both observed and full market access.	Included in the reported robustness ranges.
<i>Nonviable</i>	The specification can be solved numerically, but it violates the state-by-state positive-operating-profit condition in at least one positive-probability state.	Reported as a diagnostic boundary but excluded from the robustness ranges.
<i>No-match</i>	The specification cannot reproduce the platform R&D moment over the global search for the innovation-efficiency parameter η .	Reported as a diagnostic boundary but excluded from the robustness ranges.

The analysis contains 37 specifications—the benchmark, 27 valid alternatives, 5 nonviable cases, and 4 no-match cases. The results are comparative calibration ranges rather than confidence intervals.

B.2 Rival Entry and Commercialization

Every valid specification generates a commercial-entry threshold κ^E . This threshold is distinct from the rival-innovation threshold κ^I : the former marks the point at which entry becomes commercially viable, whereas the latter, when it exists within the feasible access range, marks the point at which rival innovation becomes privately profitable. The location of κ^E is generally stable around its benchmark value, except when the fixed entry cost F_R is varied directly.

As shown in Table B.2, excluding the alternative selections of F_R , the valid specifications place the entry threshold between 0.200 and 0.273, compared with the benchmark value of 0.234. When the observed-effective-market-access value is lowered from its measured value of 0.47 to 0.40, the calibrated entry threshold falls to 0.200. When observed access is instead set to 0.55, the threshold rises to 0.273.²⁰

¹⁹This fixed entry costs again satisfy inherited non-entry and post-DMA entry. Its midpoint is used unless itself is varied.

²⁰The observed 2 % rival market share target is held fixed in both exercises. If the same share is observed at

Other valid changes, including household substitution between digital and non-digital consumption, within-digital substitution, downstream digital dependence, platform foreign ownership, the household digital-expenditure target, the platform operating margin, the platform R&D target, and relative rival productivity, leave the entry threshold relatively close to the benchmark. However, the threshold is substantially more sensitive to the fixed entry cost F_R . Selecting the tenth percentile of the identified F_R interval lowers the entry threshold to $\kappa^E = 0.057$, while if we select the ninetieth percentile, κ^E would rise to 0.421.

The existence of a distinct commercial-entry threshold is stable, but the amount of effective market access required to cross it depends largely on a fixed cost that is only set-identified by inherited non-entry and observed post-DMA entry conditions.

Table B.2: Sensitivity of Rival Entry and Commercialization

Sensitivity input	Alternative	κ^E	Full-access rival market share (%)
Benchmark	All benchmark values	0.234	4.10
<i>Household demand, production, and ownership</i>			
Digital–non-digital substitution, σ	0.25	0.234	4.10
Digital–non-digital substitution, σ	0.70	0.234	4.10
Within-digital substitution, σ_D	6	0.232	7.82
Digital intermediate-input share, ι	0.10	0.238	4.12
Digital intermediate-input share, ι	0.20	0.237	4.11
Platform foreign ownership share, ϕ	0	0.234	4.10
Platform foreign ownership share, ϕ	0.50	0.234	4.10
Household digital-expenditure target, S_0^H	1.5%	0.234	4.10
Household digital-expenditure target, S_0^H	3.0%	0.234	4.10
<i>Market calibration and entry conditions</i>			
Inherited platform operating profit, ν_{P0}	0.30	0.236	4.58
Platform R&D-to-revenue target	6.0%	0.236	4.10
Observed rival market share target, s_R^{obs}	1.5%	0.236	3.10
Observed rival market share target, s_R^{obs}	2.5%	0.232	5.94
Observed effective market access, κ_1^{obs}	0.40	0.200	5.25
Observed effective market access, κ_1^{obs}	0.55	0.273	3.54
Inherited effective market access, κ_0	0.008	0.231	4.10
Inherited effective market access, κ_0	0.030	0.243	4.10
Relative rival productivity, A_{R0}/A_{P0}	1.00	0.236	4.10
Entry-cost selection	10th percentile	0.057	4.10
Entry-cost selection	90th percentile	0.421	4.10

Notes: All rows reported in this table are valid specifications. The full access rival market share is the expected equilibrium rival expenditure share at $\kappa_1 = 1$. Nonviable and no-match specifications are reported separately in Table B.9.

The last column of Table B.2 reports the rival’s expected market share at full market access and summarizes its commercialization, conditional on entry. The fixed entry cost largely determines whether and when the rival enters. Conditional on entry, moving F_R from the tenth to the ninetieth percentile leaves the expected full-access rival share unchanged at its benchmark value.

Post-entry commercialization is instead more sensitive to the access–share calibration and to within-digital substitution. Matching a 2 percent rival share at $\kappa_1^{\text{obs}} = 0.40$, which is below the benchmark observed-access value, produces a full-access rival market share of 5.25%. Matching

a lower level of effective market access, the calibration requires a stronger inherited relative-demand position for the rival. The rival can support entry at a lower access level. On the contrary, matching the same share at a higher observed-access value implies a weaker inherited relative-demand position and raises the entry threshold.

the same 2 percent share at $\kappa_1^{\text{obs}} = 0.55$ reduces the full-access share to 3.54%. Similarly, changing the rival market share target at the benchmark observed access $\kappa_1^{\text{obs}} = 0.47$ from 1.5% to 2.5% would expand rival market share at full access from 3.1% to 5.94%. Greater substitutability between the two digital services also strengthens the rival's commercialization response and helps it reach a 7.82 % market share at full access. The remaining valid alternatives leave the rival market share at full access unchanged or close to the benchmark.

Figure B.1 shows how the entry threshold κ_E changes across alternatives. The sensitivity results reinforce the model's distinction between entry and commercialization. The fixed entry cost primarily determines where the market shifts from Regime I to Regime II. Conditional on entry, the rival's subsequent commercialization is governed mainly by the substitutability of the two digital services and the access-share combination.

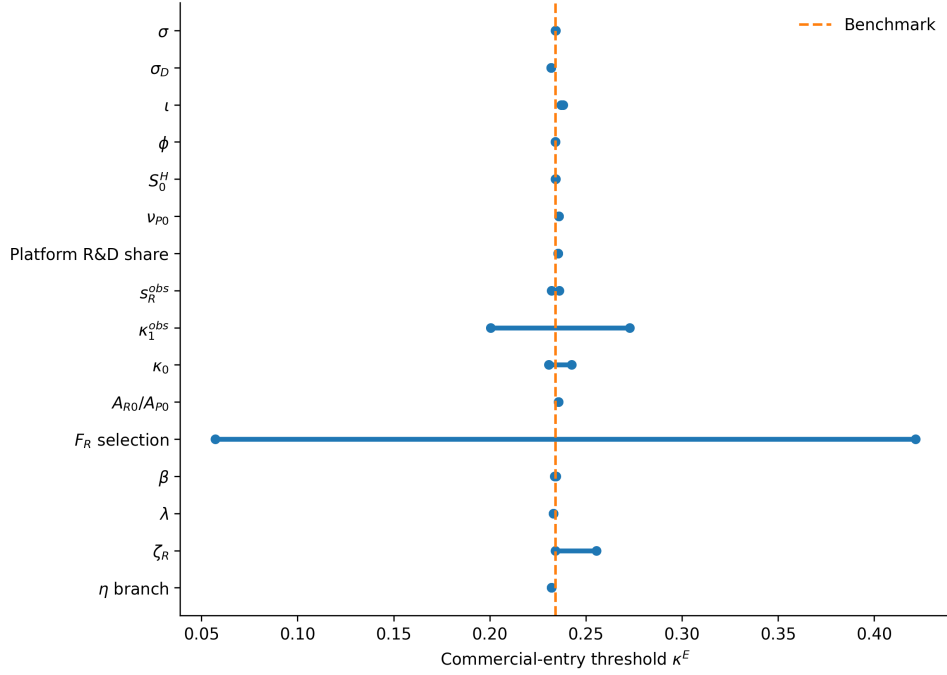


Figure B.1: **Sensitivity of the commercial-entry threshold.** Each horizontal segment reports the minimum and maximum valid threshold generated by the alternatives. The dashed vertical line denotes the benchmark. Nonviable and no-match specifications are excluded.

B.3 Innovation Responses

Innovation combines responses from both the rival and the platform. We focus on analyzing alternatives, including whether the rival crosses from Regime II into Regime III and whether the platform's innovation response remains positive.

Rival innovation. In benchmark calibration, rival remains in Regime II through full market access. But this is not universal across all alternatives—nine valid alternatives generate an interior rival-innovation threshold. And $\kappa^I > \kappa^E$ in each of these cases. Therefore, rival commercial entry might not naturally induce rival innovation.

Twenty-six out of twenty-eight valid specifications place the observed market ($\kappa = 0.47$) in Regime II, while recalibrated $\zeta_R = 2.5$ and $\zeta_R = 3$ specifications place it in Regime III.²¹

²¹This count does not include the less-disadvantaged-rival path, which does not preserve the observed 2% rival

Whether the economy enters Regime III is related to within-digital substitutability, inherited market position, rival commercial scales, household discount factor, and rival innovation capacity. Raising the within-digital elasticity to $\sigma_D = 6$ gives $\kappa^I = 0.535$. Increasing the observed rival-share target to 2.5% generates $\kappa^I = 0.843$. Raising the platform operating profit to 0.30 gives $\kappa^I = 0.892$, and matching the 2% rival market share at $\kappa_1^{\text{obs}} = 0.40$ gives $\kappa^I = 0.905$. These alternatives make rival innovation attainable when effective market access κ is higher, while leaving the observed market in Regime II.

Raising the inherited platform operating profit share from $\nu_{P0} = 0.22$ to 0.30 increases the platform productivity A_{P0} . Because the exercise holds $A_{R0}/A_{P0} = 0.95$ fixed, inherited rival productivity also rises. The resulting improvement in rival operating profitability raises the marginal return to cost ratio of disruptive innovation to a level larger than one at a sufficiently high effective market access. The rival-innovation threshold falls in the feasible range at $\kappa^I = 0.892$.

The rival-specific innovation capacity moves the innovation threshold more substantially. Actual observed market lies just below the innovation threshold at $\zeta_R = 2$ and above it at $\zeta_R = 2.5$ and 3. The upper root of the platform R&D calibration equation provides a separate branch-sensitivity check. Selecting the larger root $\eta = 207.6854$, recalibrating q_{R0} and F_R gives

$$\kappa^E = 0.232, \quad \kappa^I = 0.612.$$

Table B.3: Sensitivity Results for Rival and Platform Innovation

Alternative	κ^I	Regime at observed access	$\mathcal{R}_{RD}(1)$	$\Delta x_{PI}^{\text{obs}}$ (%)	$\Delta x_{PD}^{\text{obs}}$ (%)
Benchmark	—	Regime II	0.949	2.18	2.66
$\sigma_D = 6$	0.535	Regime II	1.612	4.19	4.88
$\nu_{P0} = 0.30$	0.892	Regime II	1.102	2.81	3.37
$s_R^{\text{obs}} = 2.5\%$	0.843	Regime II	1.152	2.69	3.29
$\kappa_1^{\text{obs}} = 0.40$	0.905	Regime II	1.087	2.18	2.66
$\beta = 0.90$	0.942	Regime II	1.052	2.00	2.42
$\zeta_R = 2.0$	0.486	Regime II	1.898	2.18	2.66
$\zeta_R = 2.5$	0.428	Regime III	2.150	2.16	2.64
$\zeta_R = 3.0$	0.390	Regime III	2.366	2.15	2.63
Upper η root	0.612	Regime II	1.537	1.58	1.89

Notes: A dash means that rival disruptive innovation remains zero boundary throughout $[\kappa_0, 1]$. $\mathcal{R}_{RD}(1)$ is the rival's disruptive marginal-benefit-to-cost ratio at zero investment and full market access. A value above one implies an interior rival-innovation threshold. Platform investment changes are measured relative to each alternative's own no-improvement equilibrium. The table reports the benchmark and all valid specifications with an interior rival innovation threshold.

Figure B.2 reports the rival's disruptive marginal-benefit-to-cost ratio at full access. Values above one imply that an interior innovation threshold exists.

Platform innovation. The positive platform innovation in both incremental and disruptive directions is not an implication of the theoretical model, but it is stable across all valid alternatives.

At the observed market, platform incremental innovation rises by 1.15 to 4.19 percent relative to each alternative's own no-improvement equilibrium, and platform disruptive investment rises market share.

Table B.4: Changes in Rival Innovation Capacity

ζ_R	κ^E	κ^I	$s_R(\kappa_1^{\text{obs}})$ (%)	$s_R(1)$ (%)	$\Pr(\xi_{RD} = 1 \mid \kappa_1^{\text{obs}})$ (%)	$\Pr(\xi_{RD} = 1 \mid 1)$ (%)
1.0	0.234	—	2.000	4.101	0.00	0.00
2.0	0.234	0.486	2.000	6.648	0.00	46.87
2.5	0.234	0.382	2.496	7.215	17.37	57.42
3.0	0.234	0.315	2.884	7.594	31.03	64.47

Notes: Unlike the recalibrated ζ_R exercises in Table B.3, this diagnostic holds q_{R0} and F_R at their benchmark values. The observed rival share is allowed to change with innovation capacity. A dash means that disruptive innovation remains zero throughout $[\kappa_0, 1]$.

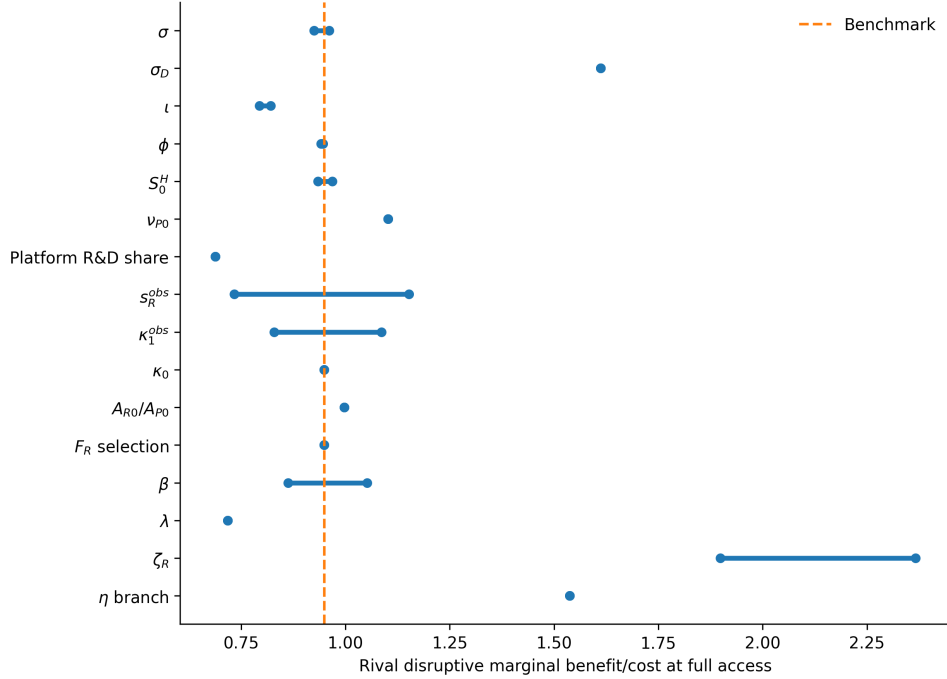


Figure B.2: **Sensitivity of rival disruptive innovation.** Each segment spans the valid values of the rival's disruptive marginal-benefit-to-cost ratio at zero investment and full market access. The dashed vertical line is the benchmark. A ratio above one implies that rival disruptive innovation becomes privately viable.

by 1.88 to 4.88 percent. At full market access, the corresponding ranges are 2.22 to 12.86 percent and 3.63 to 16.38 percent. This supports the benchmark interpretation that a decline in platform profit need not imply weaker platform innovation incentives.

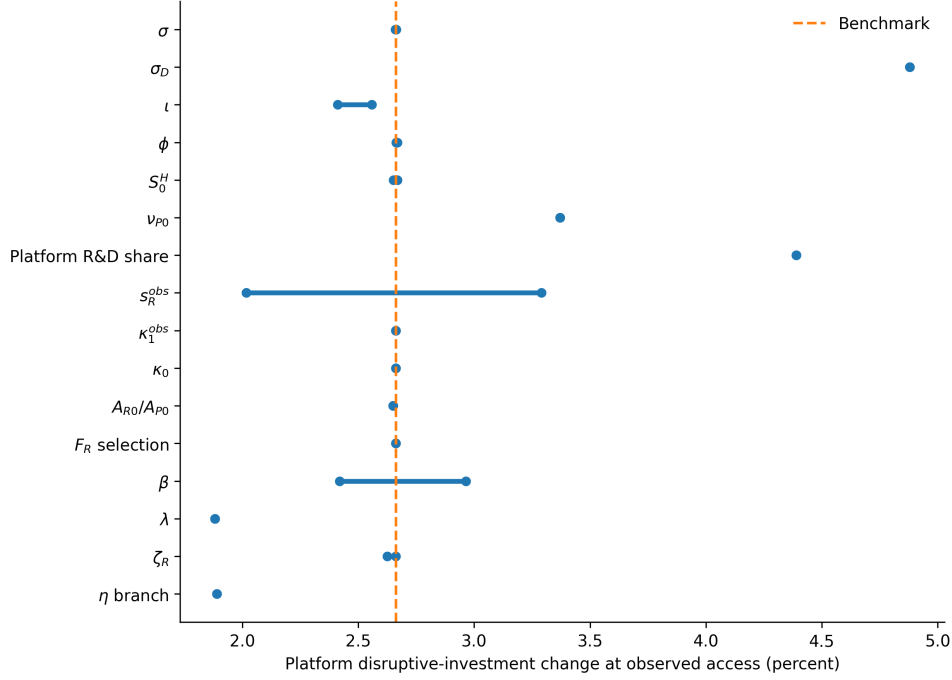


Figure B.3: **Sensitivity of platform disruptive innovation at observed access.** Each horizontal segment spans the valid percentage changes in platform disruptive investment relative to the corresponding no-improvement equilibrium. The dashed vertical line is the benchmark.

Table B.5: Sensitivity of Platform Innovation in Both Directions

Platform innovation direction	Observed access		Full access	
	Benchmark	Valid range	Benchmark	Valid range
Incremental	2.18%	[1.15, 4.19]%	4.21%	[2.22, 12.86]%
Disruptive	2.66%	[1.88, 4.88]%	5.21%	[3.63, 16.38]%

Notes: Entries report percentage changes relative to each specification's own no-improvement equilibrium. The ranges include the benchmark and all valid one-at-a-time alternatives and exclude nonviable and no-match specifications.

B.4 Welfare

Every valid specification yields a positive consumption-equivalent (CE) welfare gain at both the observed and full-market-access levels. At observed access, ΔCE ranges from 0.063% to 0.277%. And at full access, CE is increased by 0.140% to 0.857% across different specifications. The benchmark values are 0.222 and 0.470 percent, respectively.

The lower end of the welfare is driven mainly by the digital intermediate-input share ι . Reducing ι from 0.30 to 0.20 lowers the welfare gain at the observed access to 0.131%. If it is reduced further to 0.10, welfare would increase by only 0.063%. Foreign ownership directly changes domestic incidence and welfare. When $\phi = 0$, all platform profit accrues domestically.

The decrease in platform profits dampens aggregate welfare gains to 0.105% at observed access and 0.228% at full access. The upper end of the welfare is generated by $\sigma_D = 6$, in which greater within-digital substitutability strengthens commercialization, activates rival innovation, and raises the full-access welfare gain to 0.857%.

The rival's market position substantially affects welfare. At observed access, if we vary the rival market share target from 1.5% to 2.5%, the welfare gains would change from 0.167% to 0.277%. On the contrary, the entry cost does not affect welfare gains as largely as it affects the entry threshold—the tenth- and ninetieth-percentile F_R selections produce observed-access welfare gains of 0.238 and 0.207 percent, respectively.

Table B.6: Sensitivity Results for Welfare

Alternative	CE at observed access (%)	CE at full access (%)
Benchmark	0.222	0.470
$\iota = 0.10$	0.063	0.140
$\iota = 0.20$	0.131	0.285
$\phi = 0$	0.105	0.228
$\phi = 0.50$	0.172	0.366
$s_R^{\text{obs}} = 1.5\%$	0.167	0.358
$s_R^{\text{obs}} = 2.5\%$	0.277	0.656
$\sigma_D = 6$	0.237	0.857
F_R : 10th percentile	0.238	0.485
F_R : 90th percentile	0.207	0.454
Upper η root	0.194	0.583

Notes: Consumption-equivalent welfare changes are measured relative to each specification's own no-improvement equilibrium. All rows are valid specifications. The ϕ alternatives only change the domestic incidence of platform profit without changing the underlying commercialization mechanism.

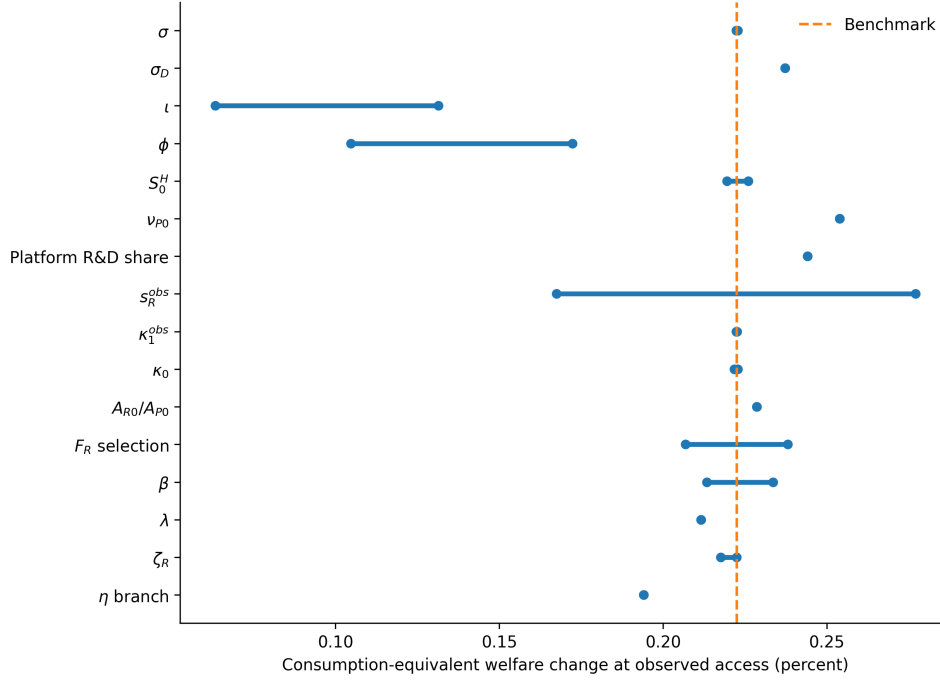


Figure B.4: **Sensitivity of welfare at observed access.** Each horizontal segment spans the valid consumption-equivalent welfare changes produced by the alternatives for the reported input. The dashed vertical line is the benchmark.

B.5 Robustness of the Diagnostic Exercises

Alternative Paths for Narrowing Inherited Rival Gaps. The main less-disadvantaged-rival path closes the benchmark quality and productivity gaps by the same fraction. To test whether the results depend on that same ratio restriction, I close the quality gap and the productivity gap separately while holding all remaining benchmark primitives fixed.

Table B.7: Sensitivity to the Path Used to Narrow Inherited Rival Gaps

Path	$\rho^{I,FA}$	$\rho^{I,obs}$	$\rho^{E,0}$	ρ^{50}
Quality gap only	0.0153	0.2307	—	0.9473
Productivity gap only	—	—	—	—
Equal fractional closure of both gaps	0.0151	0.2262	0.9450	0.9440

Notes: $\rho^{I,FA}$ and $\rho^{I,obs}$ are the fractions of the specified benchmark gap(s) that must be closed for disruptive innovation to become viable at full and observed market access. $\rho^{E,0}$ is the fraction at which entry becomes viable already at inherited access. ρ^{50} is the fraction at which the expected rival share reaches 50 percent at full access. A dash means the outcome is not reached on $[0, 1]$.

The quality-only path is close to the main joint path, which I implement and report in the main text. Closing 1.53% of the benchmark quality gap (with productivity gap fixed) would incentivize the rival to innovate at full market access, and closing 23.07% would let the rival start innovation at the current observed market. By contrast, closing the entire benchmark productivity gap while holding the quality gap fixed does not make rival innovation viable throughout the valid range of effective market access. This path does not support rival entry at the inherited stage nor generate 50 percent rival market share at full access.

These asymmetric results show that the equal fractional closure path is dominated by the

closure of the relative-demand gap rather than the productivity gap. In fact, these reflect the benchmark calibration: the inherited quality gap is large while the rival productivity reaches 95% of the platform's. It does not support a general claim that quality always matters more than productivity.

Figure B.5 varies the two gap-closure fractions independently and classifies the observed market. The main path is the diagonal $\rho_q = \rho_A$. Across the viable region, the Regime-II/Regime-III boundary is nearly vertical: closing roughly one-quarter of the benchmark quality gap is sufficient to activate rival innovation at observed access, whereas productivity-gap closure alone is not. The gray area identifies combinations with strong demand but insufficient productivity that violate the positive operating-profit condition.

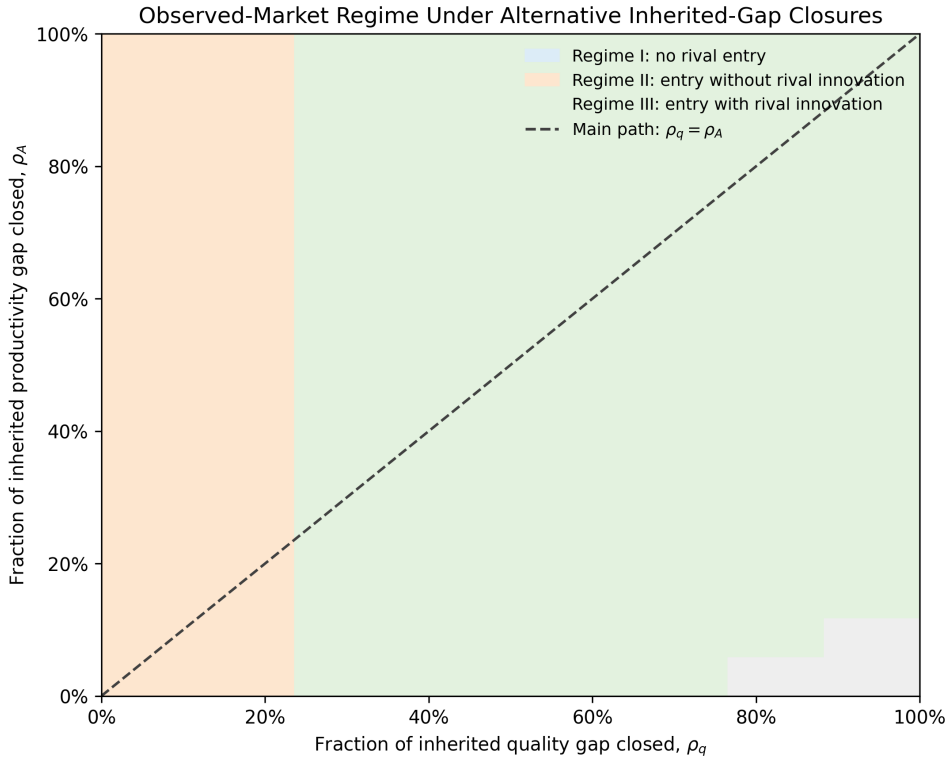


Figure B.5: **Observed-market regimes under alternative inherited- gap closures.** The horizontal and vertical axes independently vary the fractions of the benchmark quality and productivity gaps that are closed. The dashed diagonal is the main less-disadvantaged-rival path. Gray cells are operationally nonviable. Regime I does not appear among viable cells because observed market access already supports entry at the benchmark point.

Sensitivity of the Share-Gap Accounting Diagnostic The benchmark accounting decomposition assigns 19.4 percent of the log-odds distance to the 50–50 reference to the remaining market-access term and 80.6 percent to the inherited relative-demand term. To test its robustness, I vary the observed rival share from 1.5 to 2.5 percent and observed access from 0.40 to 0.55.

Across different combinations of observed rival market share and observed access, the market-access component accounts for the gap from 14.3% to 25.0%, corresponding to the contribution from the relative-demand component 85.7% to 78.1%. Therefore, the 19.4/80.6 split is measurement-dependent. But we can also see from Table B.8 that the qualitative

Table B.8: Sensitivity of the Share-Gap Accounting Decomposition

Observed rival share (%)	Observed market access	Market-access component (%)	Relative-demand component (%)
1.5	0.40	21.9	78.1
1.5	0.47	18.0	82.0
1.5	0.55	14.3	85.7
2.0	0.40	23.5	76.5
2.0	0.47	19.4	80.6
2.0	0.55	15.4	84.6
2.5	0.40	25.0	75.0
2.5	0.47	20.6	79.4
2.5	0.55	16.3	83.7

Notes: Components are shares of the total log-odds distance from the observed rival share to the 50–50 reference.

accounting result is stable. The relative-demand component represents the majority of the observed log-odds distance to the 50–50 reference.

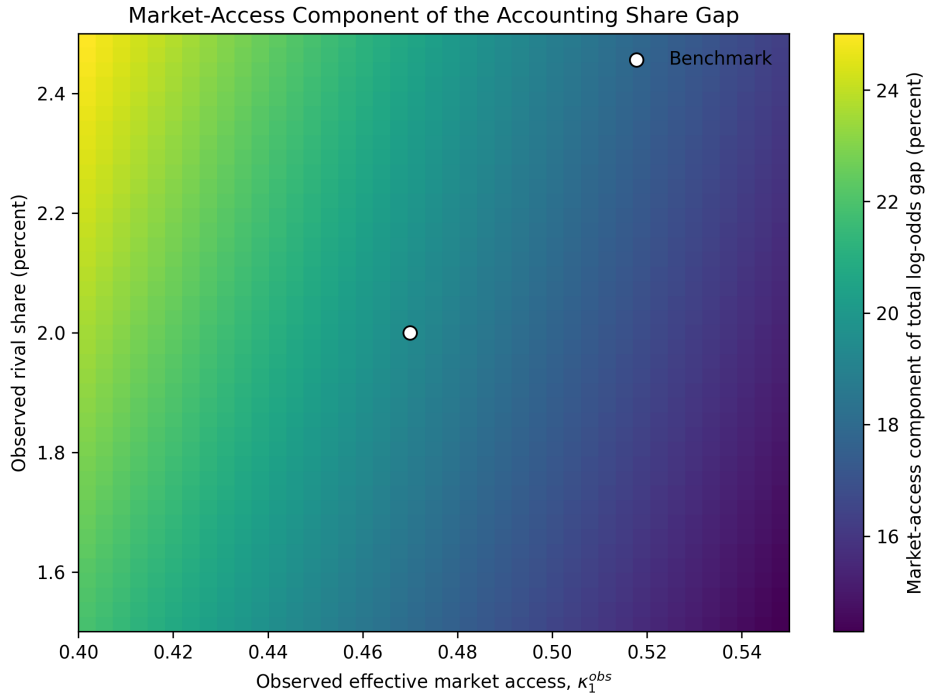


Figure B.6: **Sensitivity of the market-access component of the share-gap decomposition.** The color scale reports the percentage of the total log-odds distance to the 50–50 reference represented by the current market-access term. The point marks the benchmark pair $(s_R^{obs}, \kappa_1^{obs}) = (0.02, 0.47)$.

Calibration Boundaries and Numerical Validation The failed alternatives help identify the admissible quantitative domain. Four specifications cannot match the platform R&D-to-revenue target over the global η search interval. Five specifications match the calibration moments but violate operational viability in at least one positive-probability state.

Table B.9: Calibration and Operational-Viability Boundaries

Input	Value	Status	Diagnostic interpretation
Platform R&D share	10.0%	No match	The platform R&D-to-revenue moment cannot be matched on the global η search interval.
λ	0.90	No match	The platform R&D-to-revenue moment cannot be matched on the global η search interval.
δ	0.08	No match	The platform R&D-to-revenue moment cannot be matched on the global η search interval.
γ_D	2.0	No match	The platform R&D-to-revenue moment cannot be matched on the global η search interval.
σ_D	2	Nonviable	The rival has a negative operating profit in at least one positive-probability state (minimum profit -0.003).
ν_{P0}	0.18	Nonviable	The rival has a negative operating profit in at least one positive-probability state (minimum profit -0.043).
A_{R0}/A_{P0}	0.90	Nonviable	The rival has a negative operating profit in at least one positive-probability state (minimum profit -0.047).
δ	0.18	Nonviable	The rival has a negative operating profit in at least one positive-probability state (minimum profit -0.077).
γ_D	3.0	Nonviable	The rival has a negative operating profit in at least one positive-probability state (minimum profit -0.018).

The maximum innovation best-response error is 5.26×10^{-12} . Solving the innovation game at observed and full access produces a maximum strategy deviation of 9.02×10^{-12} . The maximum labor-market residual is 4.44×10^{-16} . The smallest rival operating profit among valid specifications is 0.00738. Even though these diagnostics do not establish global uniqueness of the strategic equilibrium, they show that the reported branch is numerically stable in the sensitivity analysis.

Appendix C Measuring Effective Contestability

This appendix summarizes the effective-contestability measure used in the quantitative analysis. The complete methodology, indicator definitions, scoring manual, source documentation, and validation exercises are presented in the companion contestability paper (Cheng et al., 2026).

The unit of assessment is a particular core platform service, gatekeeper, jurisdiction, and date. For the present application, it is app distribution on Apple devices in the European Union. The framework distinguishes between *de jure contestability*, denoted by (κ_t^J) , and *de facto contestability*, denoted by (κ_t^F) . The former captures the extent to which laws, regulations, enforceable commitments, and formal platform obligations establish opportunities for rivals to compete. The latter captures the extent to which those opportunities can be exercised in practice.

Both layers evaluate four dimensions: user mobility and switching; rival access and distribution; platform conduct and neutrality; and data, interoperability, and ecosystem access. Each dimension is scored on a normalized scale from zero to one using paired *de jure* and *de facto*

indicators. The benchmark assigns equal weight to the four dimensions. A score of zero represents the absence of the relevant formal or practically usable opportunity, while a score of one represents the corresponding full-contestability benchmark. Effective contestability combines the two layers multiplicatively:

$$\kappa_t = \kappa_t^J \kappa_t^F. \quad (\text{C.1})$$

The product reflects complementarity between formal protection and practical implementation. Comprehensive formal obligations create little effective access if challengers cannot exercise them, while favorable practices provide a less secure competitive opportunity if they are not supported by binding requirements. Unlike an arithmetic average, the product does not allow a high score in one layer to compensate mechanically for a severe shortfall in the other.

The index excludes entry, market shares, revenue, profits, and innovation. These variables are equilibrium outcomes that the model seeks to explain. For example, the installation procedure facing users may inform the de facto score, but the challenger’s resulting installation or expenditure share does not. This separation avoids using the same outcome both to construct contestability and to calibrate or evaluate the model.

Table C.1 reports the aggregate values used in the application.

Table C.1: Effective Contestability in EU App Distribution on iOS

Market state	(κ_t^J)	(κ_t^F)	(κ_t)
Pre-compliance baseline	(0.18)	(0.08)	(0.0144)
Post-compliance observation	(1.00)	(0.47)	(0.4700)

The post-compliance de jure score of one means that formal obligations address the access channels included in the index; it does not imply that every aspect of app-store competition is perfectly regulated. The de facto score of (0.47) captures the substantial practical frictions that remain after formal coverage is established.

In the model, the dominant platform’s accessibility is normalized to one, and the measured effective contestability enters directly as the challenger’s relative access weight. Let $(\tilde{P}_{jt} = P_j/q_{jt})$ denote supplier (j)’s quality-adjusted price. Conditional on challenger entry, the digital price index is

$$P_{Dt} = \left[\tilde{P}_{Pt}^{1-\sigma_D} + \kappa_t \tilde{P}_{Rt}^{1-\sigma_D} \right]^{\frac{1}{1-\sigma_D}}. \quad (\text{C.2})$$

Equivalently, the coefficient on the challenger term in the primal CES composite is (κ_t^{1/σ_D}) . The challenger’s expenditure-share odds are

$$\frac{s_{Rt}}{1 - s_{Rt}} = \kappa_t \left(\frac{\tilde{P}_{Rt}}{\tilde{P}_{Pt}} \right)^{1-\sigma_D}. \quad (\text{C.3})$$

Thus, (κ_t) determines relative commercial accessibility, but it does not determine entry or market share on its own. Those outcomes also depend on prices, quality, productivity, and entry costs. The equality between the measured index and the model’s access weight is a normalization that avoids introducing an unidentified transformation between them.

Finally, the observed values of (κ_t) describe market states rather than regulatory choices. The counterfactual exercise varies effective contestability to compare equilibrium outcomes

across alternative access environments; it does not estimate a policy-response function linking particular legal obligations to (κ_t) .

Appendix D Quantitative Application

Although the institutional setting is motivated by the DMA, the quantitative exercise addresses the broader class of conduct-based ex ante regulations that seek to improve market access and effective contestability without directly determining entry or market structure.

I apply the model to mobile app distribution on Apple devices in the European Union for three reasons. First, the importance of mobile app distribution extends beyond the consumption of digital services. Burley and Fradkin (2025) estimate that physical goods and services accounted for \$1.014 trillion, or 78 percent, of the billings and sales facilitated by the global App Store ecosystem in 2024. In Europe, physical goods and services accounted for \$109 billion of the estimated \$148 billion total. These transactions span general retail, travel, food delivery, groceries, and ride-hailing. This evidence motivates the model’s treatment of digital services as an input into non-digital economic activity: changes in the accessibility, quality, and innovation of app-mediated services can affect production and commercialization well beyond the digital sector.

Second, the market provides a particularly clear mapping between the model and its empirical application. The dominant platform corresponds to the incumbent app-distribution channel, while the rival represents an alternative app store or distribution provider. Effective contestability captures the rival’s practical ability to reach users, and the model’s entry, commercialization, quality, productivity, and innovation decisions have direct counterparts in the app-distribution market. The use of digital services by the non-digital sector captures, in reduced form, the role of app distribution in delivering productive digital tools to businesses.

Third, mobile app distribution provides comparatively rich quantitative evidence. We can combine longitudinal information on market outcomes, transactions, and usage, commission structures, developer activity, challenger entry, platform financial performance, and firm-level research and development with the evidence collected through the contestability analysis. Input-output data and evidence on business use of digital services can further discipline the strength of the linkage between digital innovation and non-digital production. Although market-specific innovation expenditures and the exact magnitude of this linkage remain imperfectly observed, the available evidence permits a more disciplined quantitative application than is currently possible for many other digital markets.